Shear Force and Bending Moment Diagrams

- A ***BEAM*** is a long, slender, structural member designed to support transverse loadings.
- A transverse loading is applied perpendicular to the axis of the beam.
- Beams classified by their supports.

Need to know the internal shear force and the internal bending moment at all locations in the beam. This information is usually presented as a graph or diagram of these values vs. position.

- Bending Moment
- The algebric sum of the moments of the forces on either side of the section of a loaded beam is called Bending Moment.
- SHEAR FORCE
- The algebric sum of the vertical forces on either side of the section of a loaded beam is called Shearing Force

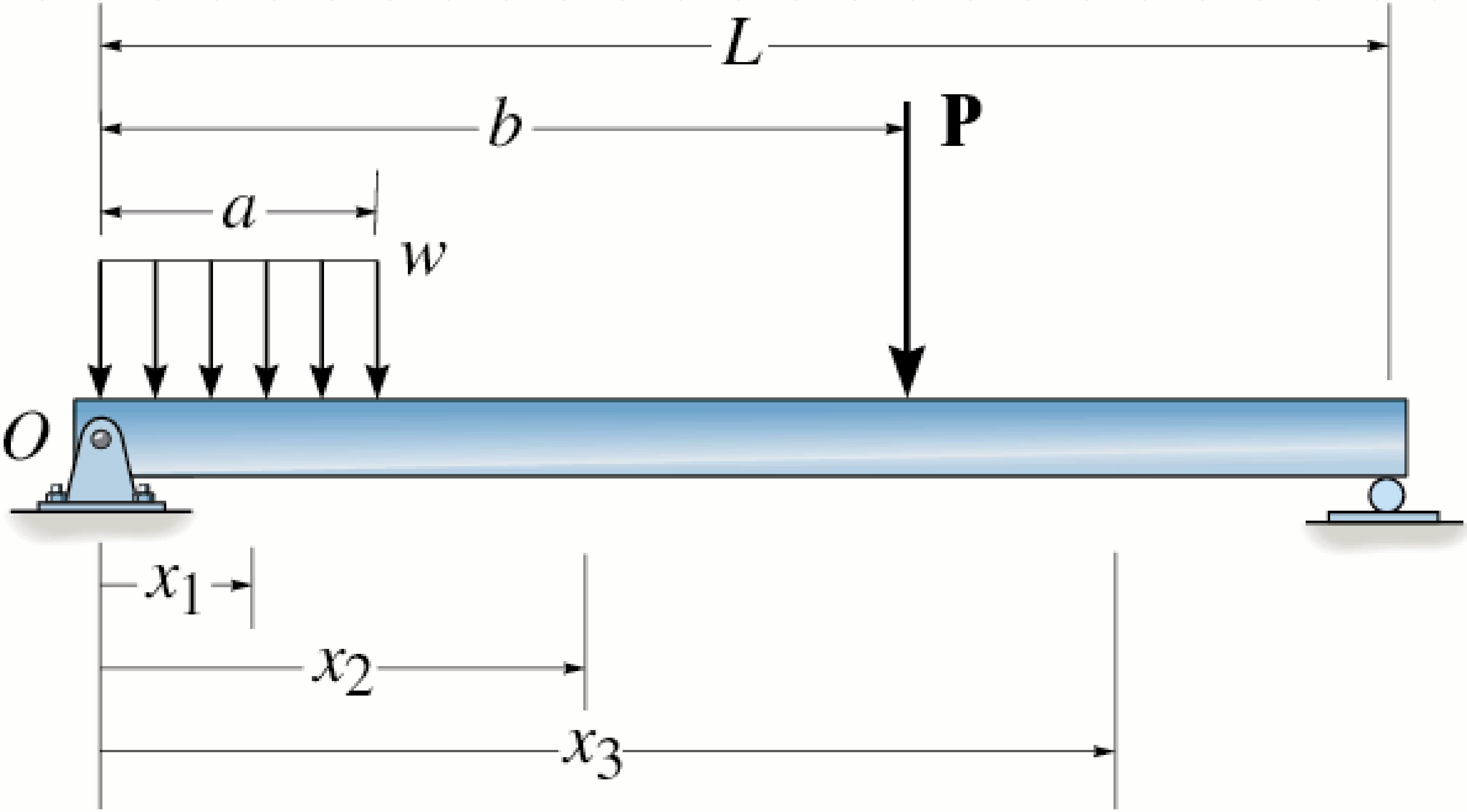
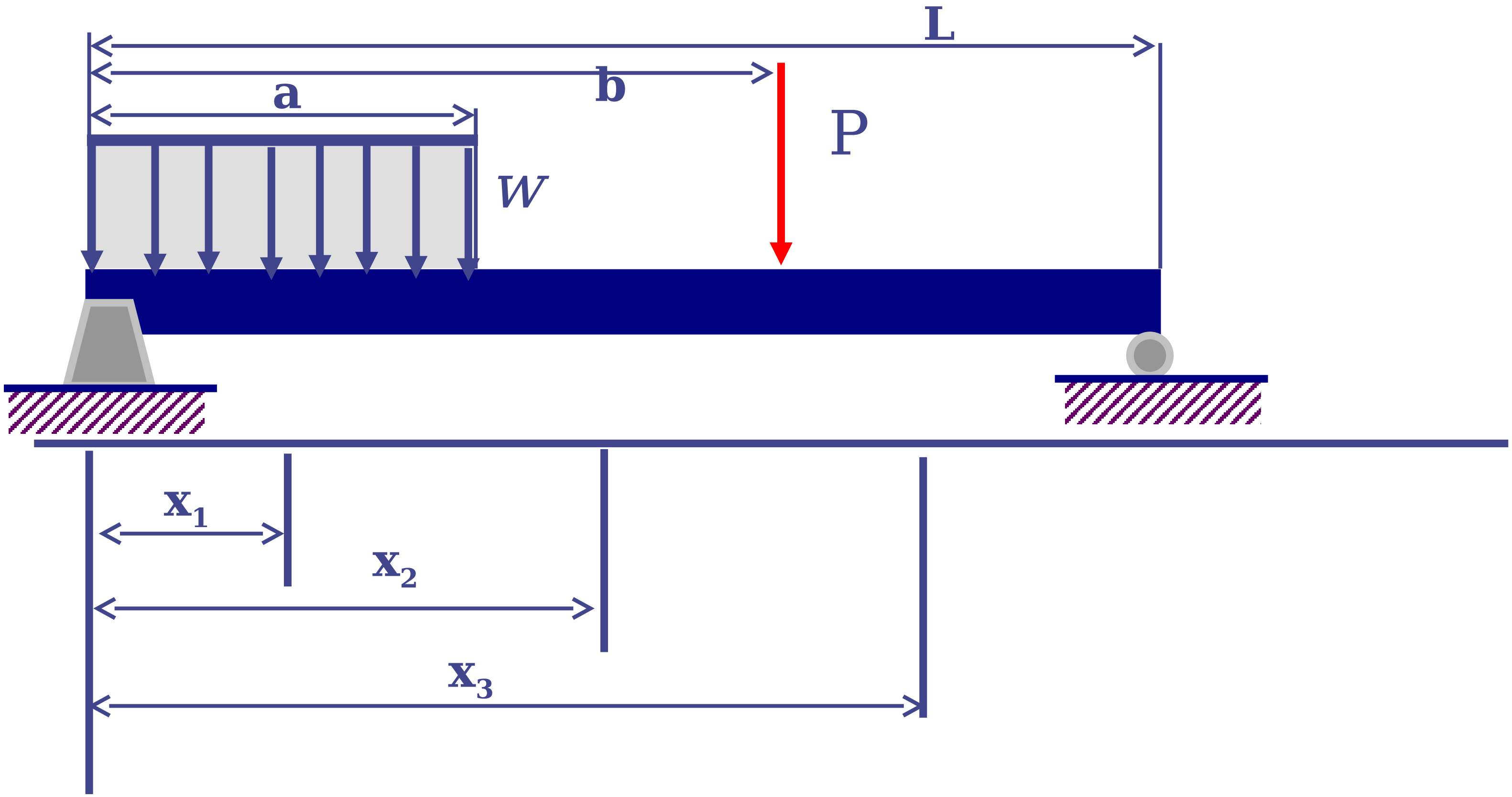
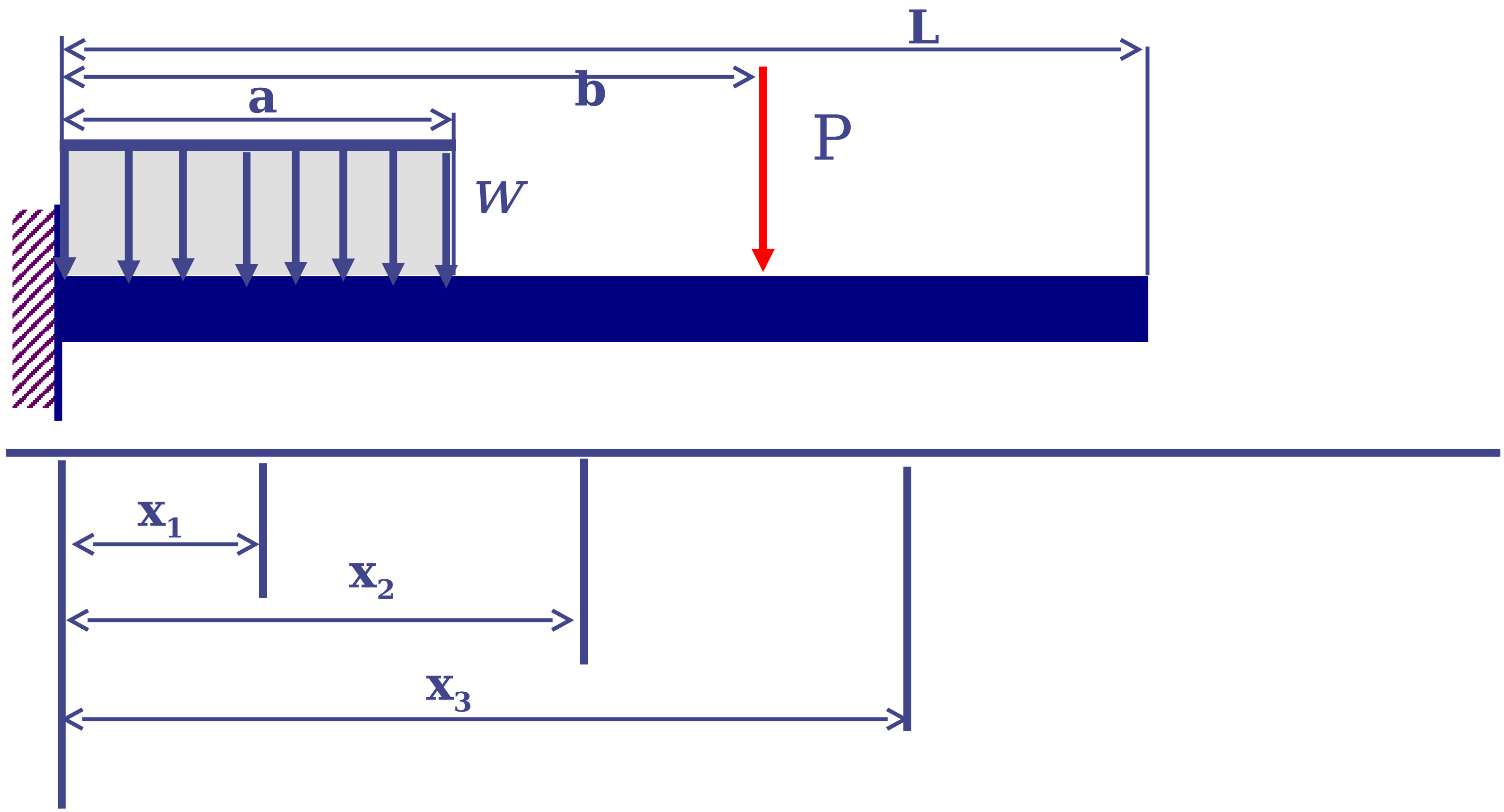


Figure 07.10



Simply Supported Beam



Cantilever Beam

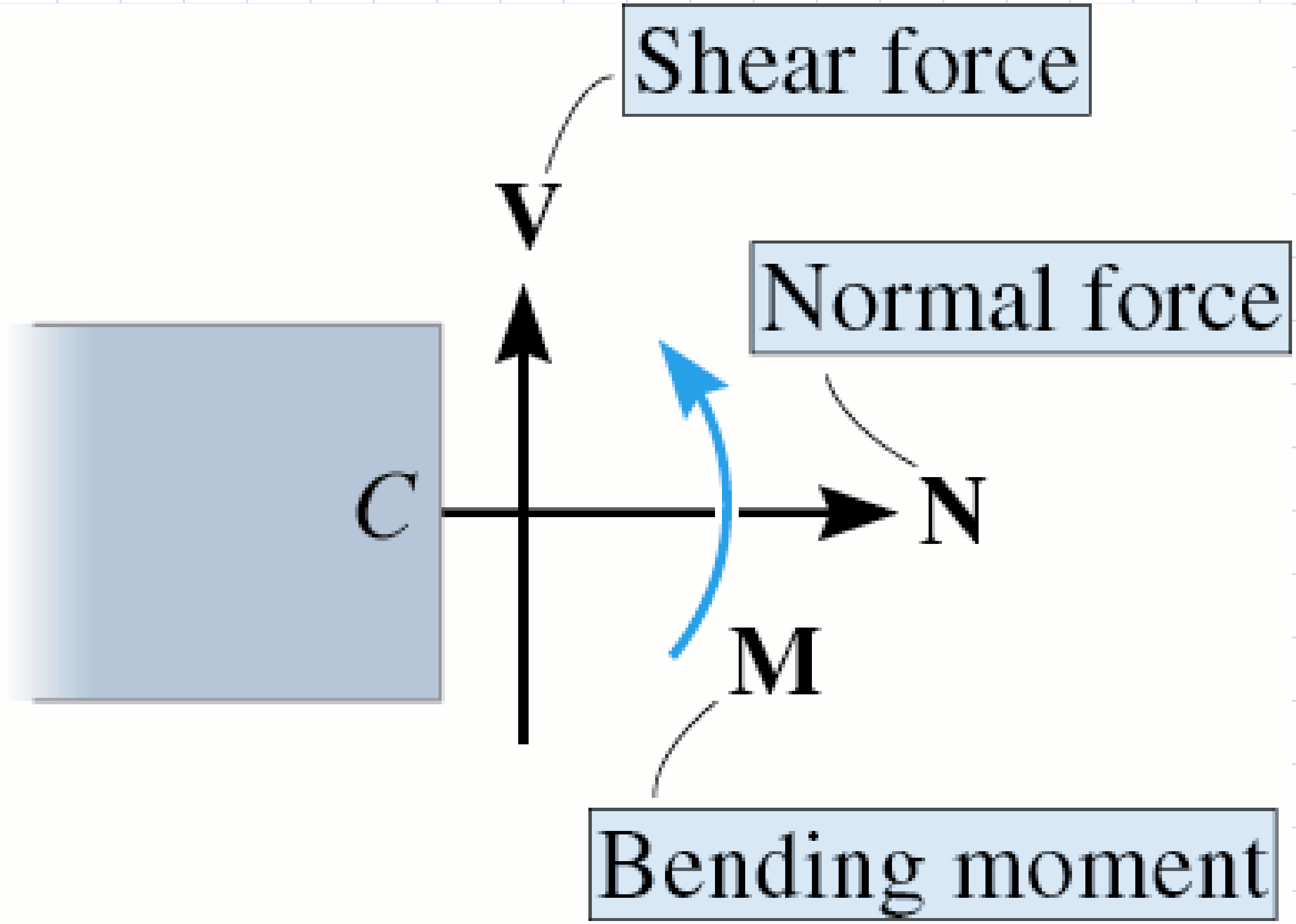
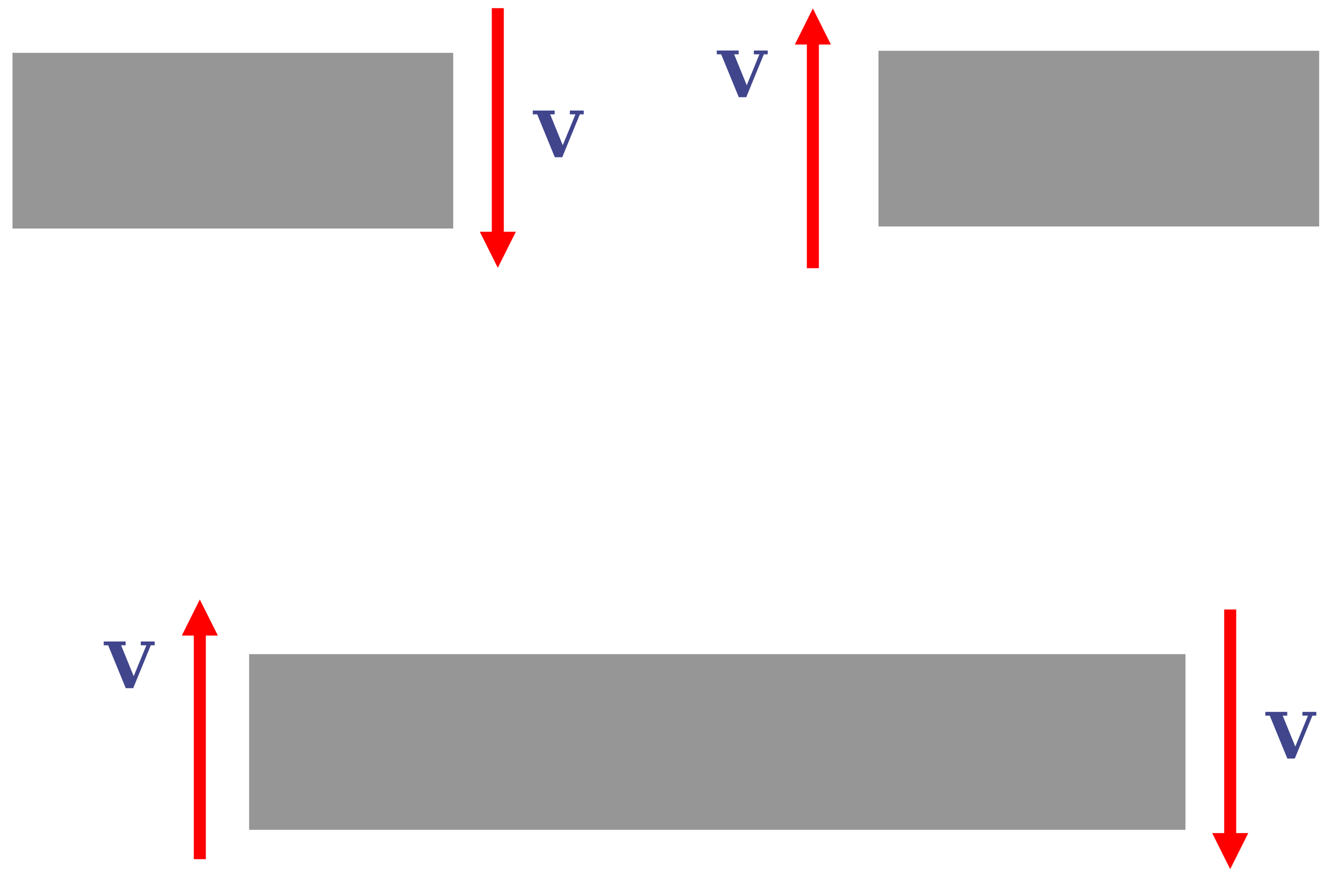
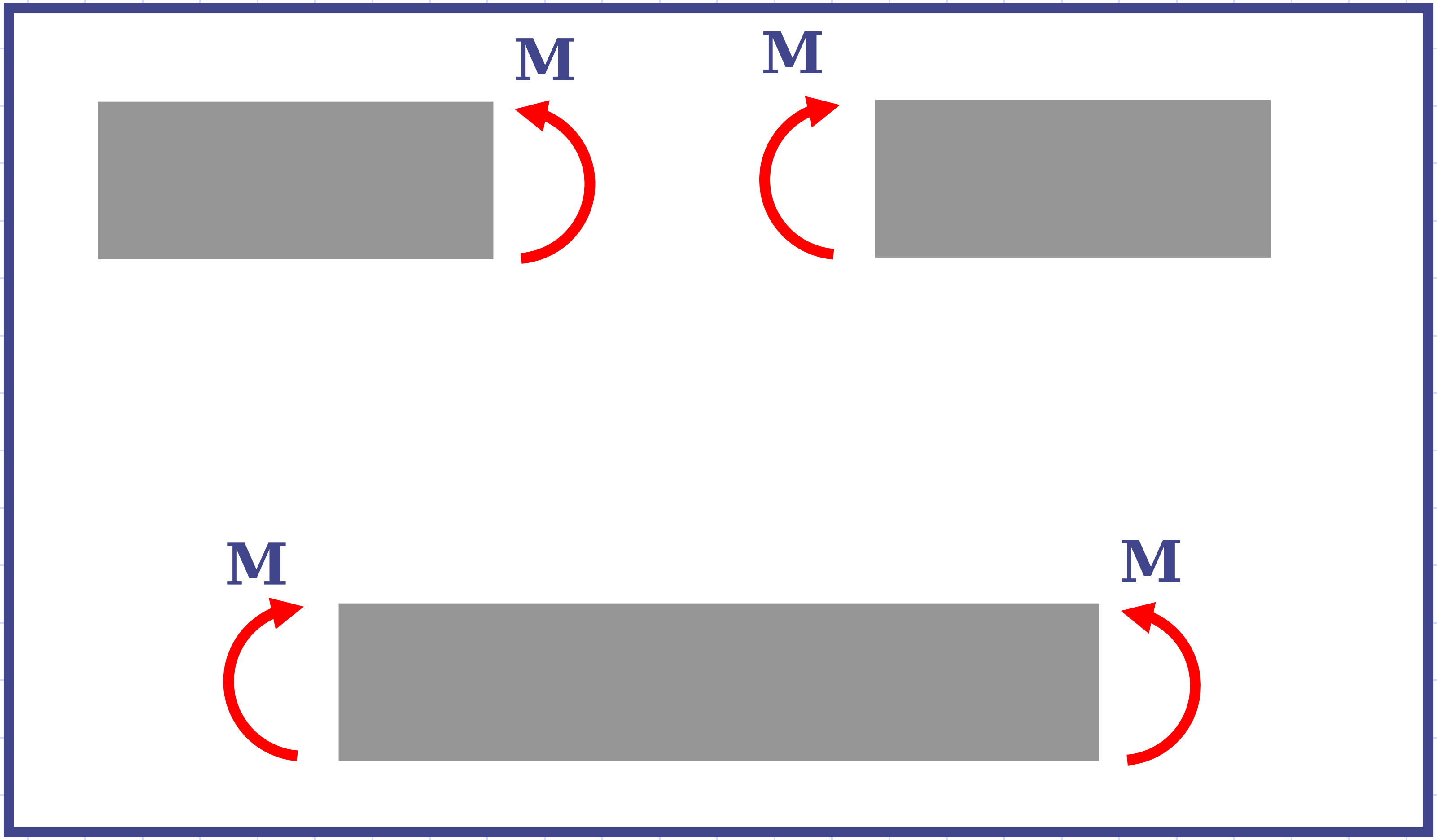


Figure 07.02(a)

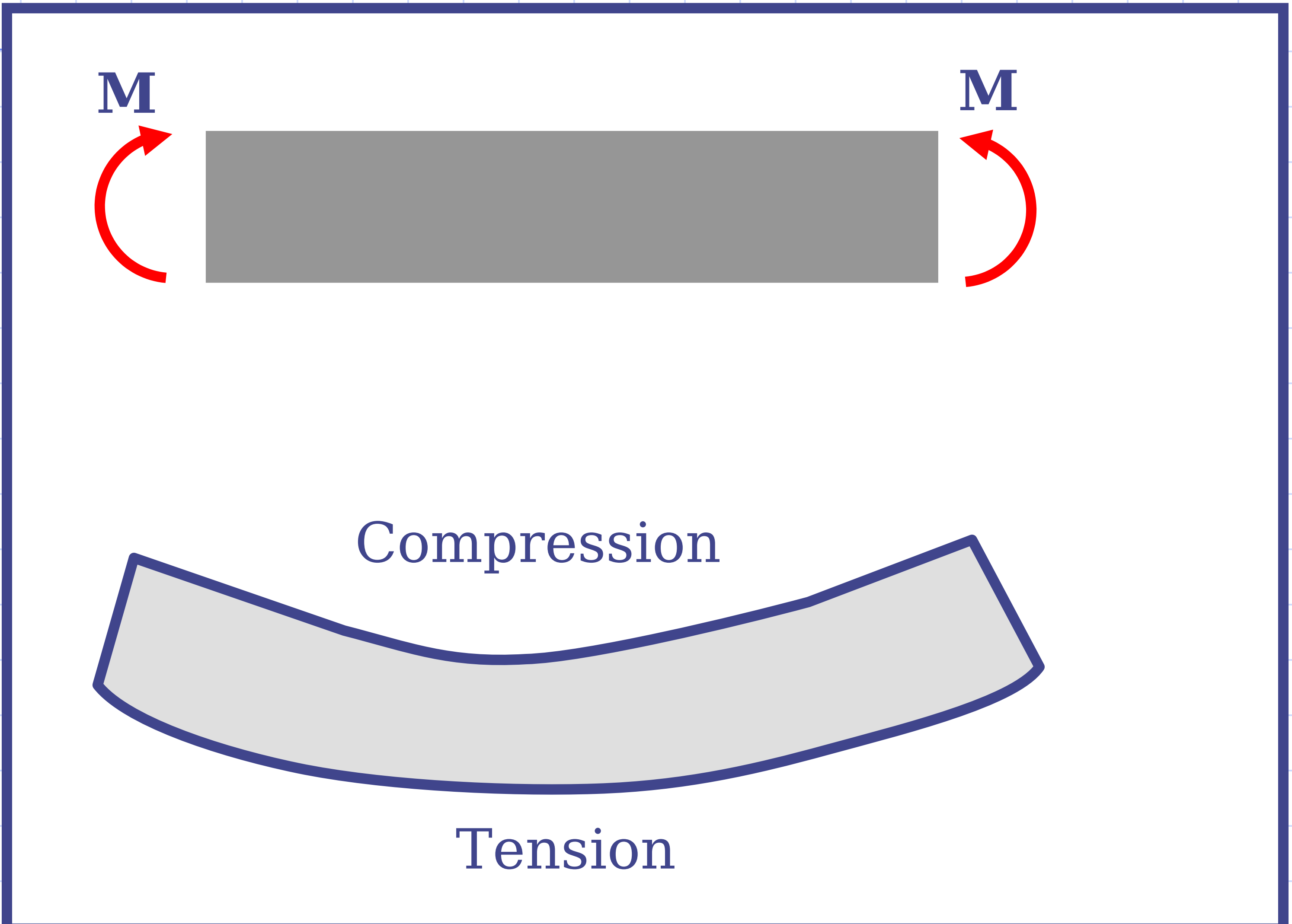
Positive Shear

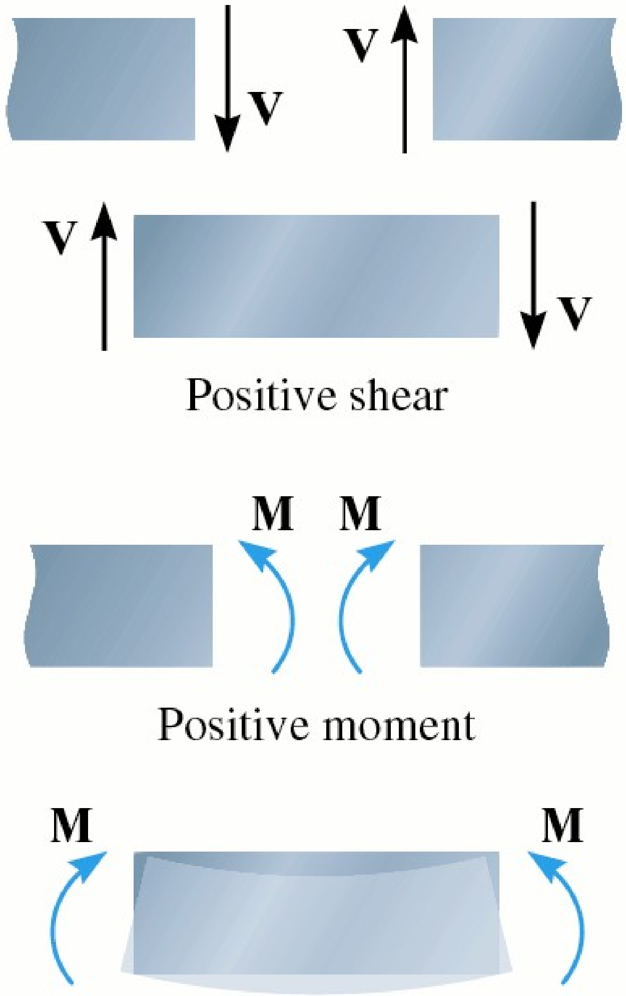


Positive Moment



Positive Moment

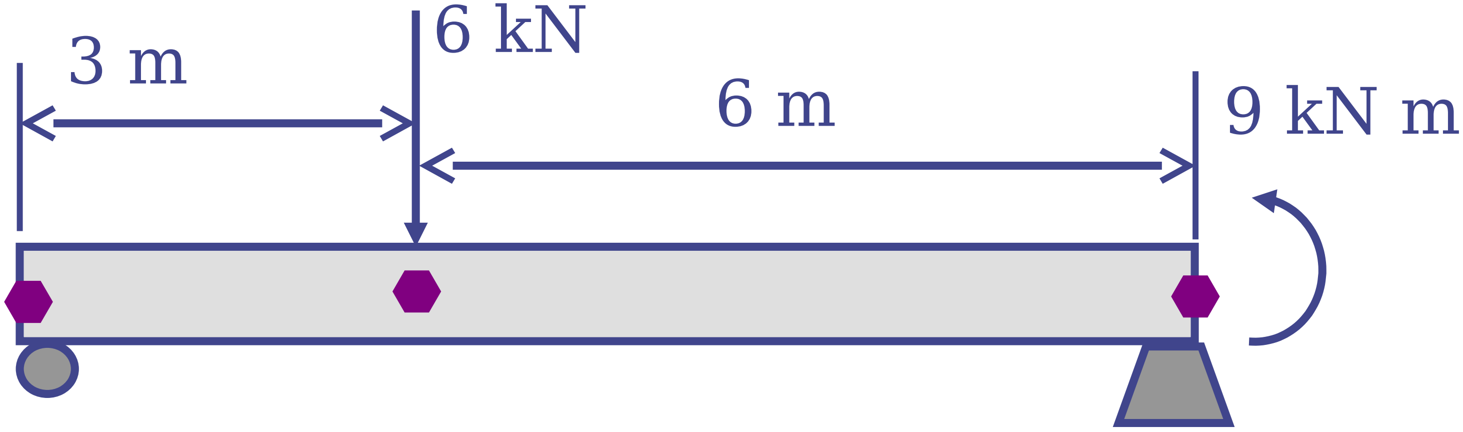




Beam sign convention

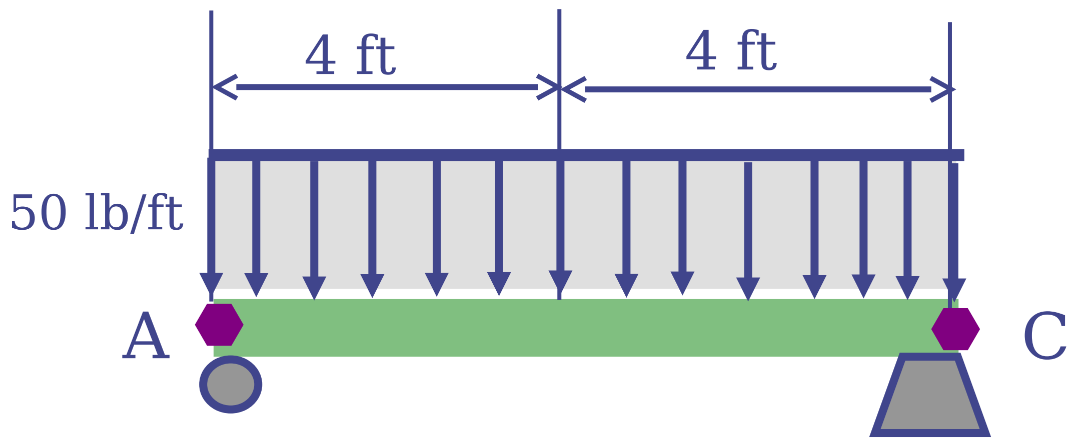
Figure 07.11

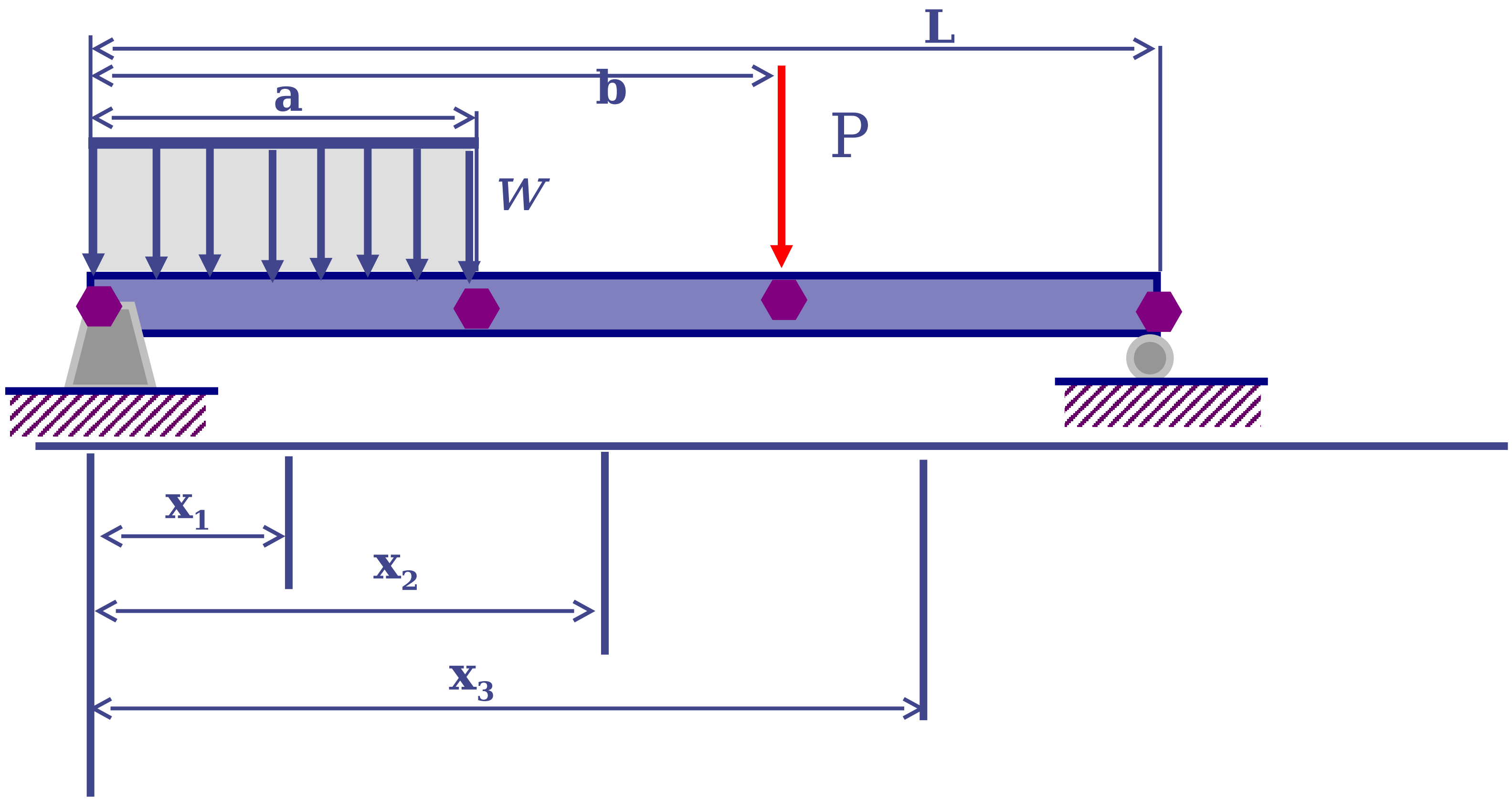
Critical Points



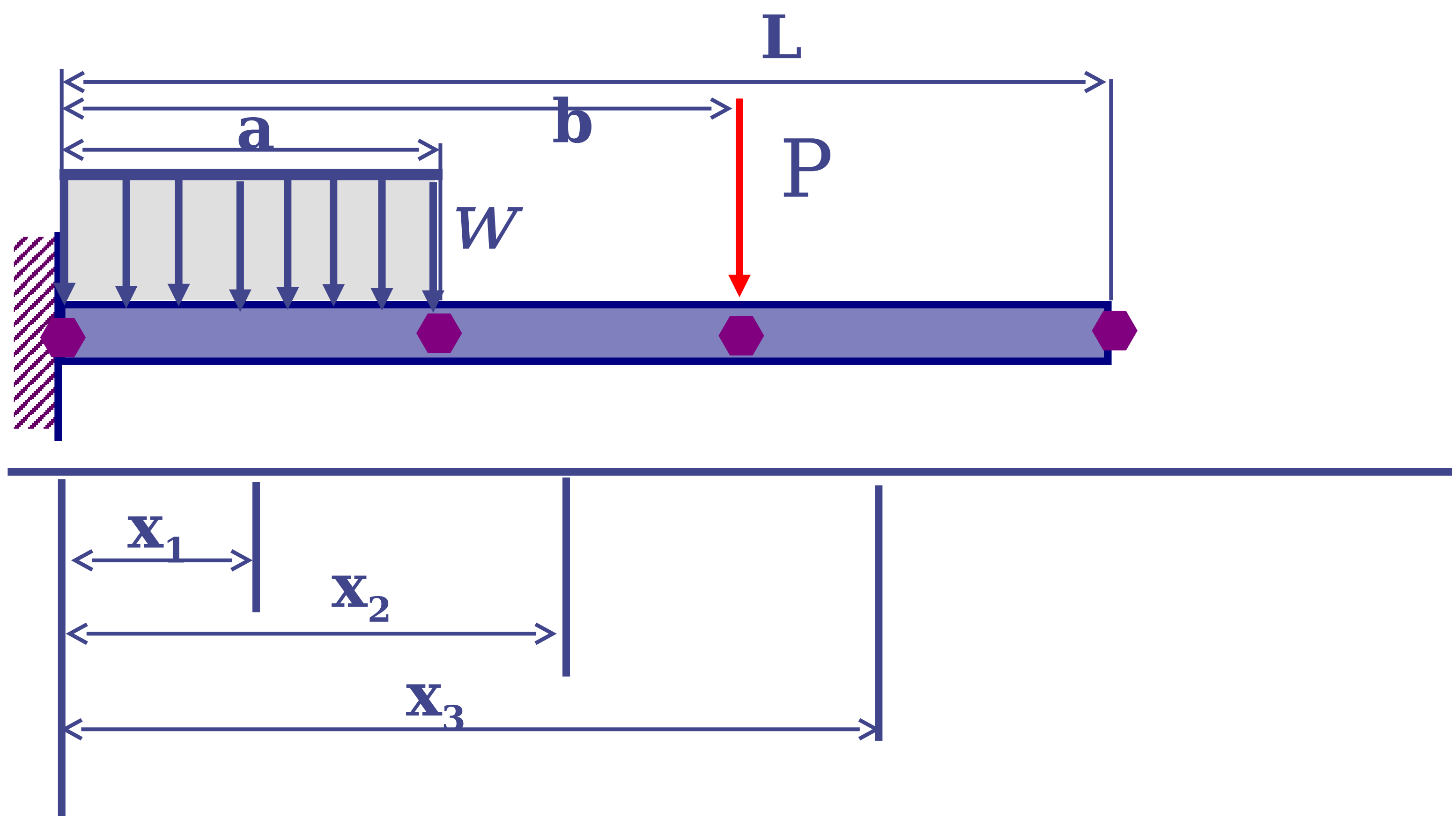
Determine the internal forces just to the left and the right of the external force

Critical Points



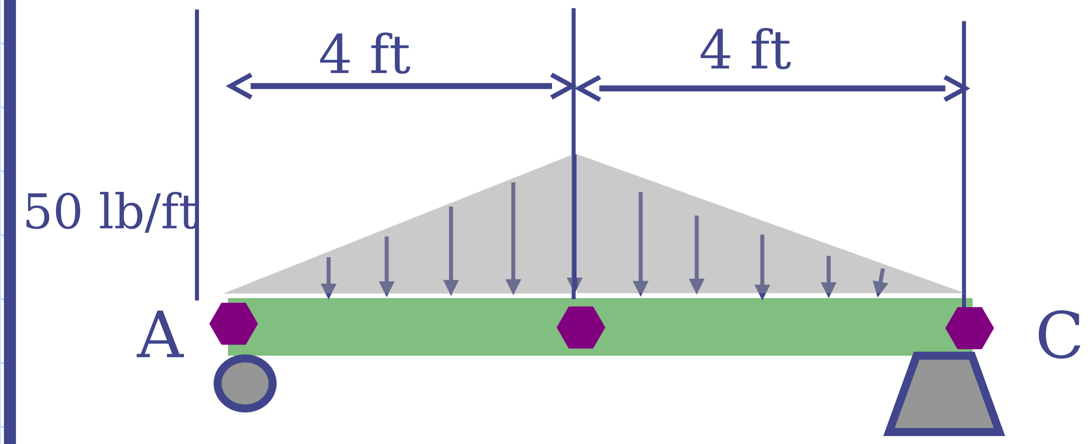


L
b
a
P
W
x1
x2
x3

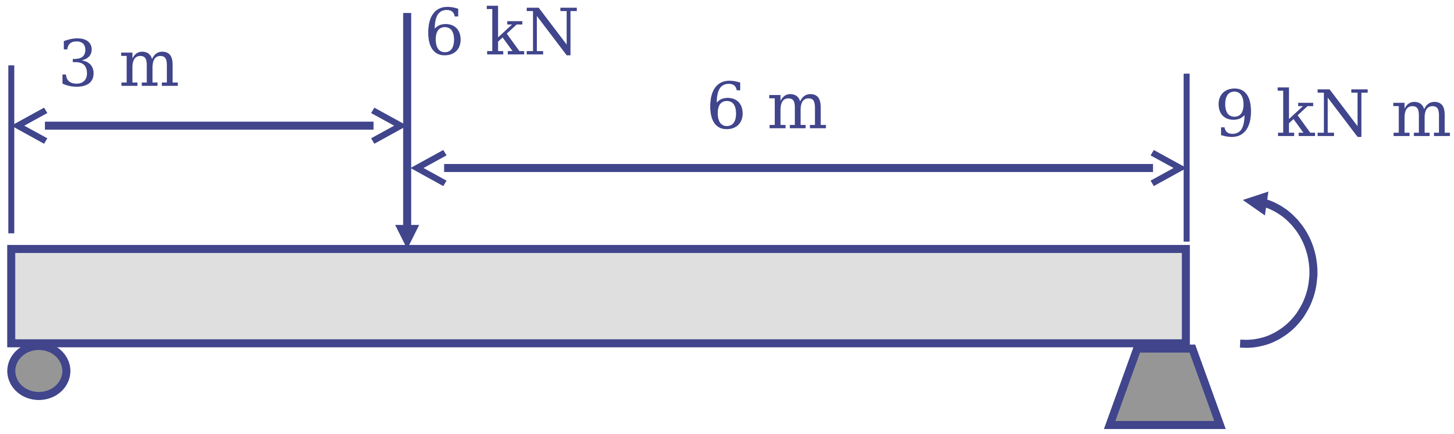


L
b
a
P
w
x1
x2
x3

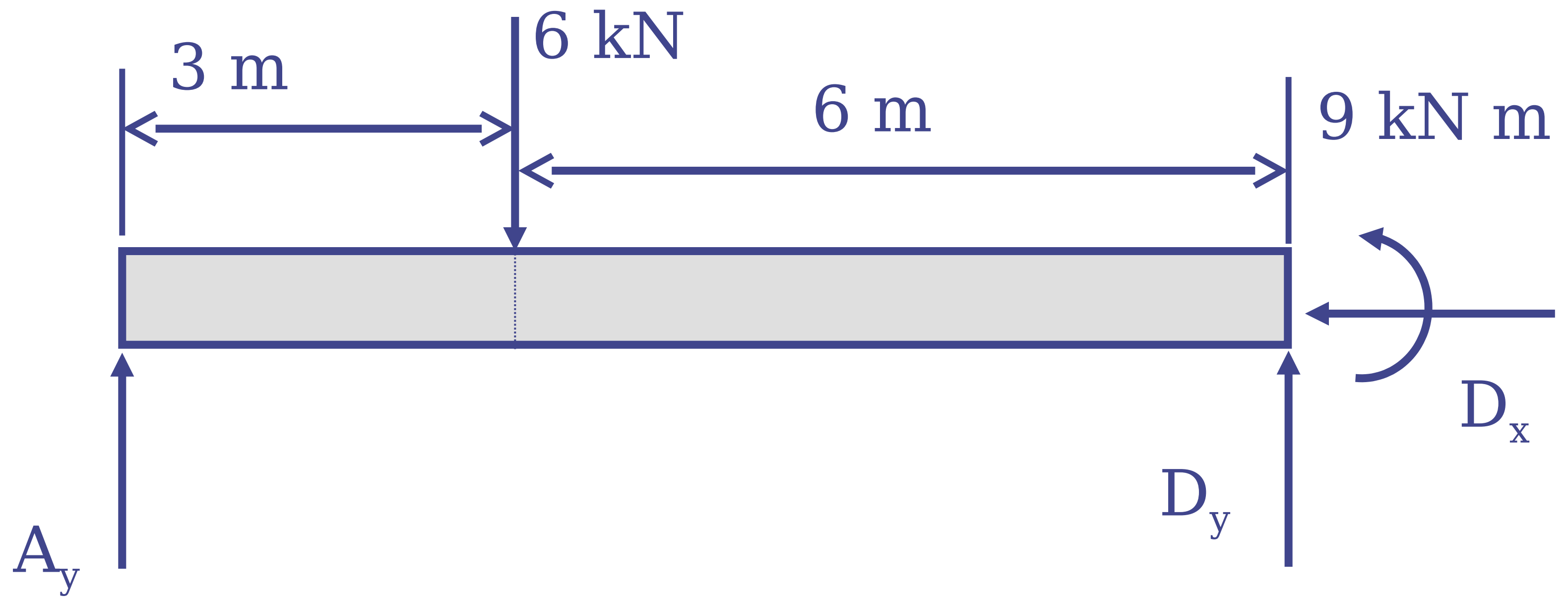
Critical Points



Shear Force and Bending Moment Diagram



FBD of Beam



$$\sum F_x = 0$$

$$D_x = 0$$

$$\sum F_y = 0$$

$$A_y - 6 + D_y = 0$$

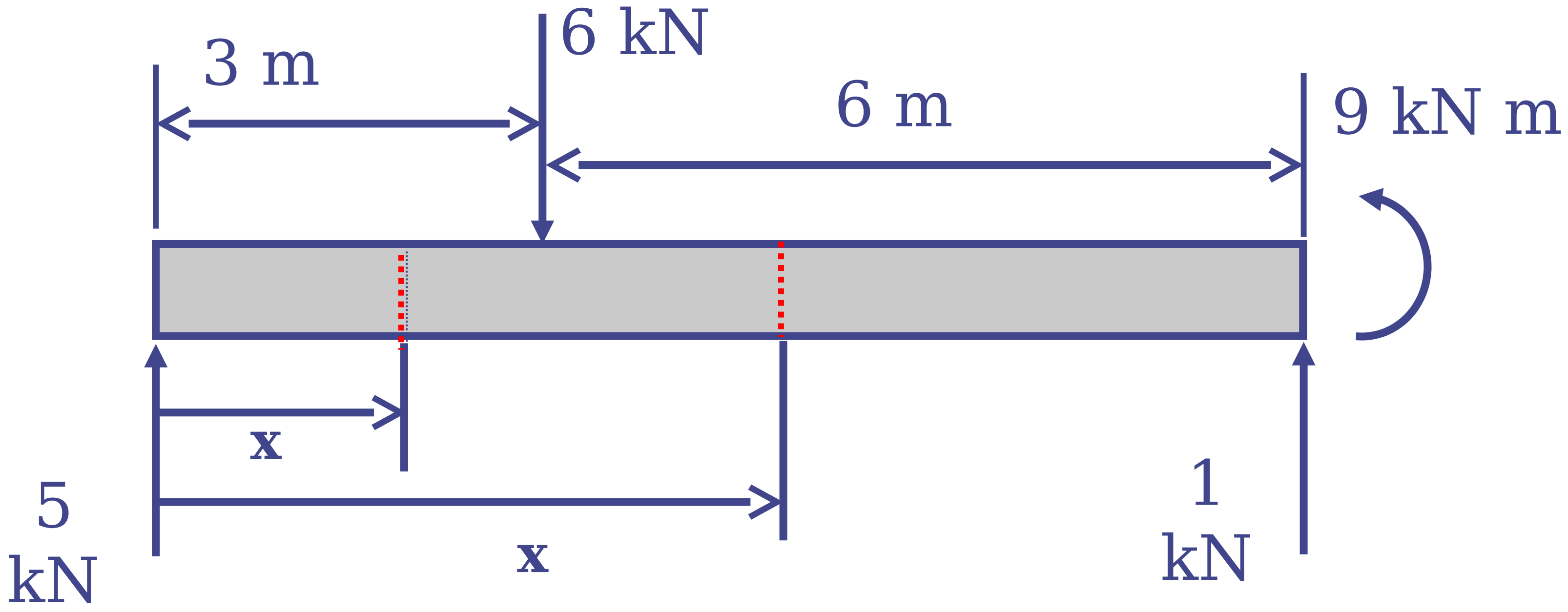
$$\sum M_D = 0$$

$$9 + 6(6) - A_y(9) = 0$$

$$A_y = 5\,kN$$

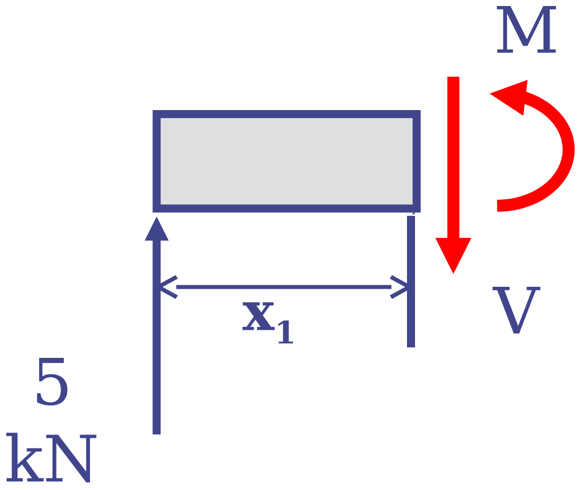
$$D_y = 1\,kN$$

FBD of Beam



$$0 \leq x < 3$$

$$3 < x \leq 9$$



$$0 \le x < 3$$

$$\sum F_y = 0$$

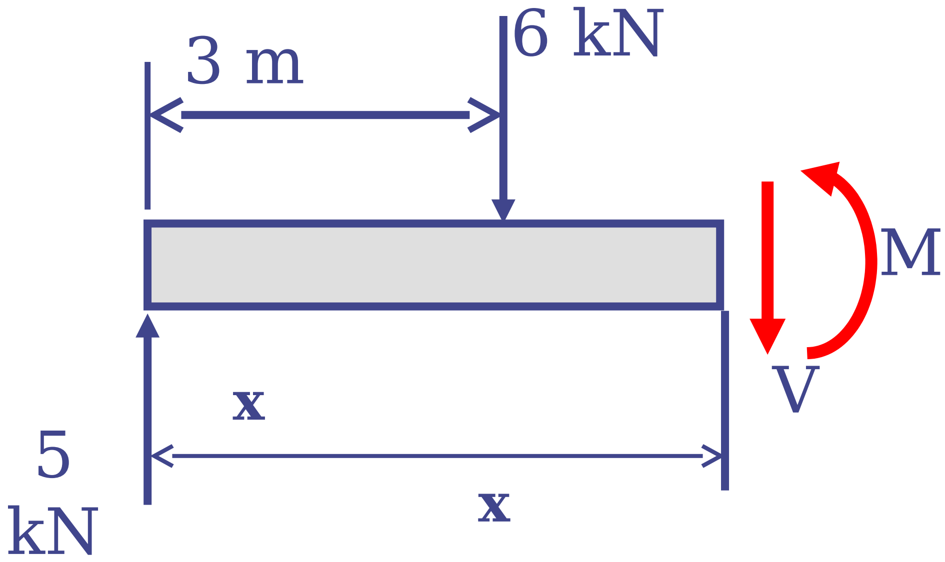
$$5 - V = 0$$

$$V = 5\ \mathrm{kN}$$

$$\sum M = 0$$

$$M - (5\ \mathrm{kN})x = 0$$

$$M = 5x\ \mathrm{kN \cdot m}$$



$$3 < x \le 9$$

$$\sum F_y = 0$$

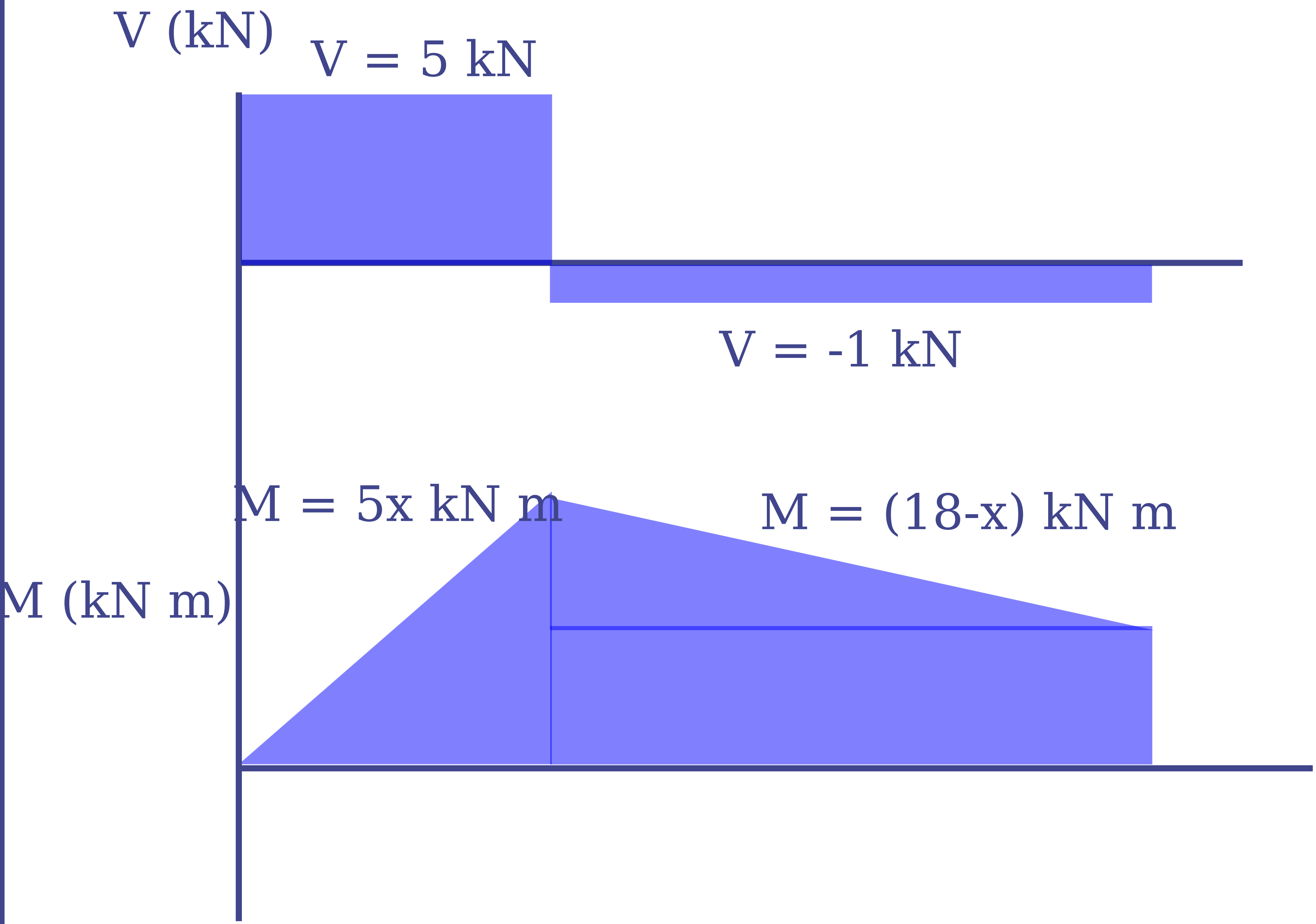
$$5\ \text{kN} - 6\ \text{kN} - V = 0$$

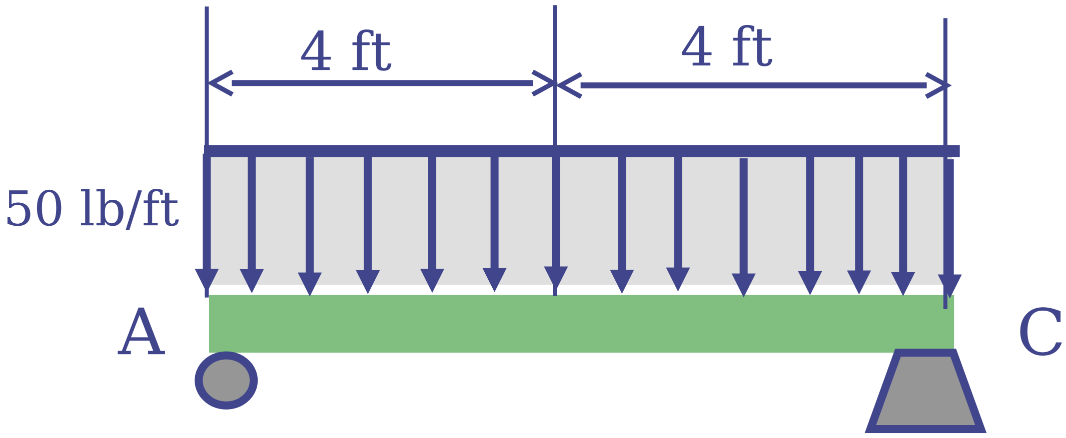
$$V = -1\ \text{kN}$$

$$\sum M = 0$$

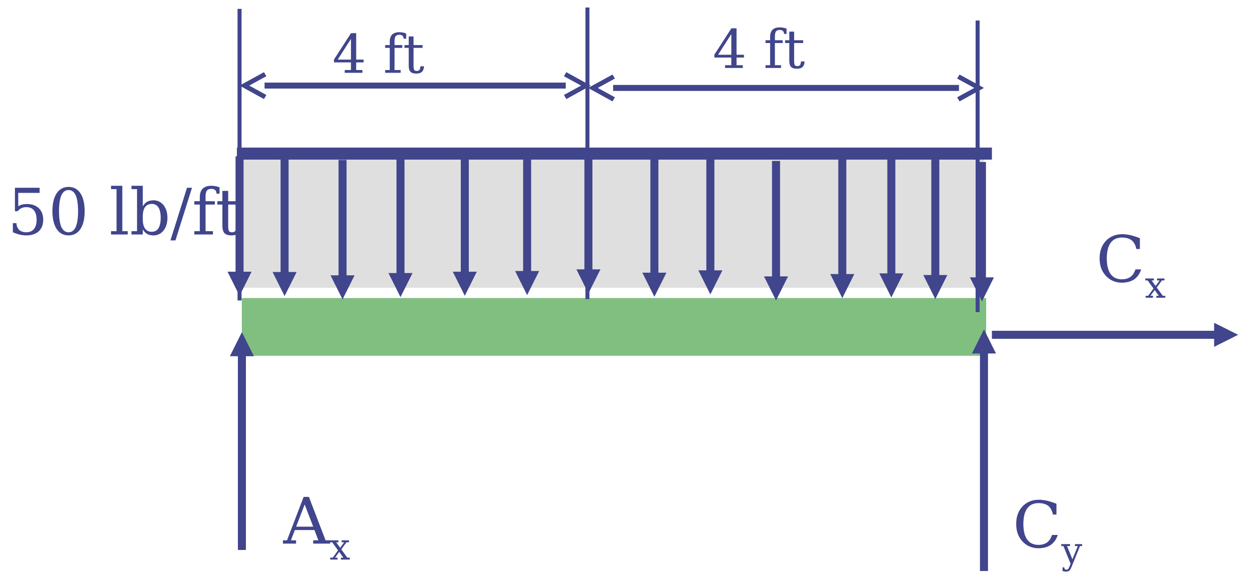
$$M - (5)x + (6)(x - 3) = 0$$

$$M = (18 - x)\ \text{kN}\cdot\text{m}$$

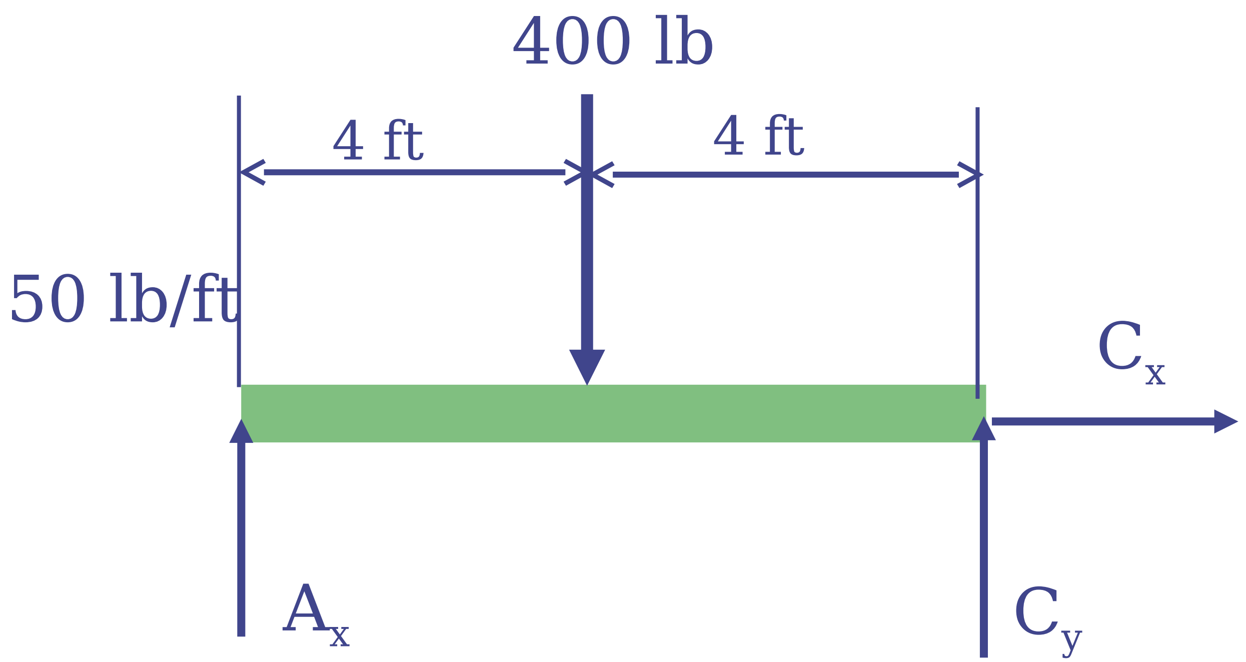

V (kN)
V = 5 kN
V = -1 kN
M = 5x kN m
M = (18-x) kN m
M (kN m)



4 ft
4 ft
50 lb/ft
A
C



4 ft
4 ft
50 lb/ft
C_x
A_x
C_y



400 lb
4 ft
4 ft
50 lb/ft
Cx
Ax
Cy

$$\sum F_x = 0$$

$$C_x = 0$$

$$\sum F_y = 0$$

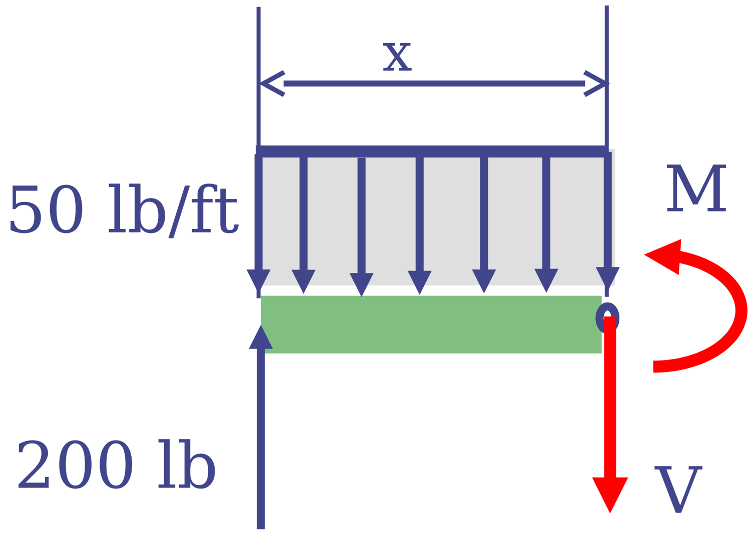
$$A_y + C_y - 50(8) = 0$$

$$\sum M_A = 0$$

$$50(8)(4) - C_y(8) = 0$$

$$A_y = 200 \text{ lb}$$

$$C_y = 200 \text{ lb}$$



$$0 \le x \le 8$$

$$\sum F_y = 0$$

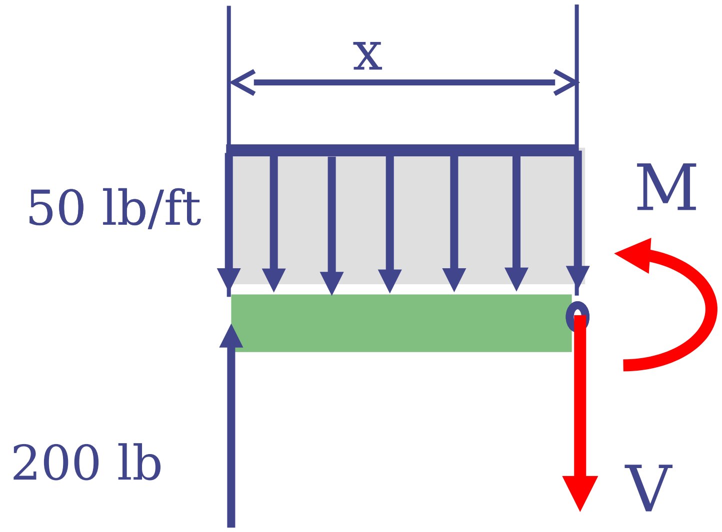
$$-V + 200\ \text{lb} - 50x = 0$$

$$V = (50x - 200)\ \text{lb}$$

$$\sum M = 0$$

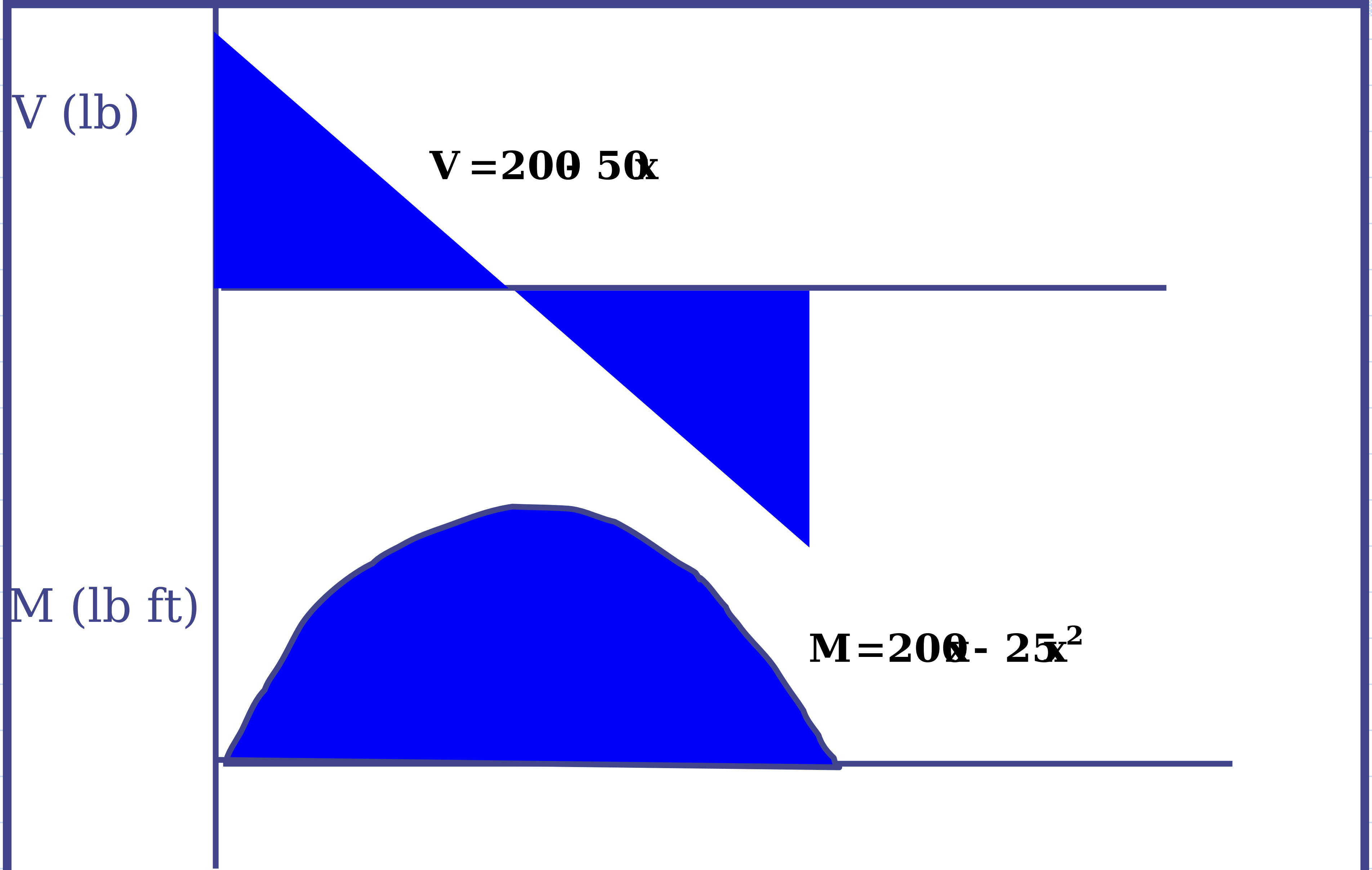
$$M - 200x + 50(x)\left(\frac{x}{2}\right) = 0$$

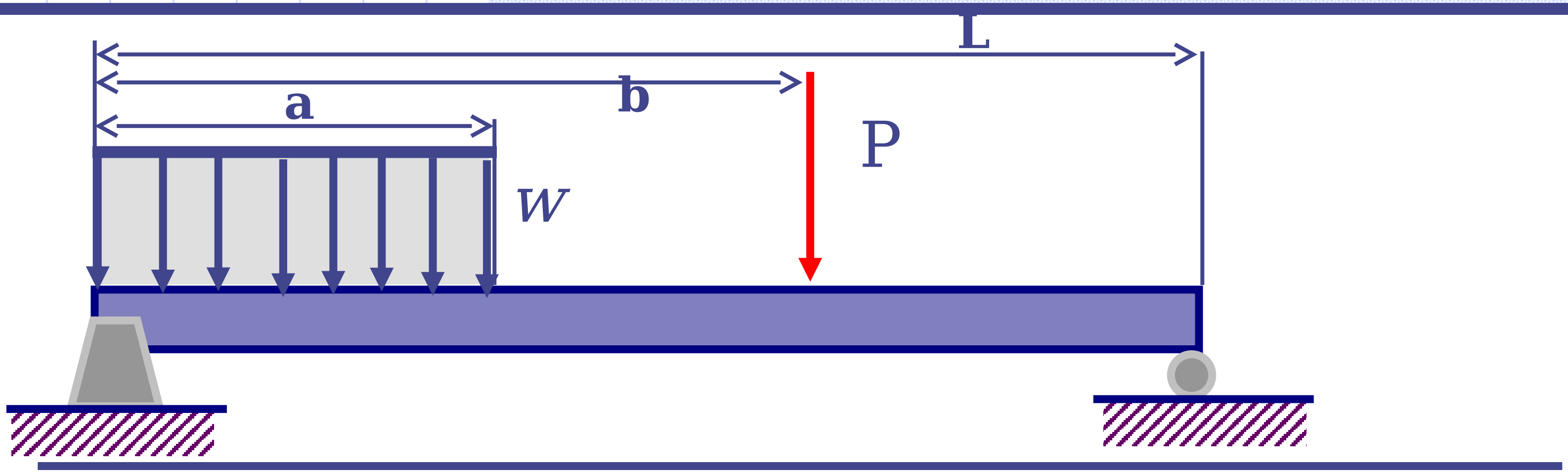
$$M = 200x - 25x^2\ \text{lb} \cdot \text{ft}$$



$at\,x = 0, \quad V = 200\,lb \quad M = 0$

$at\,x = 8, \quad V = -200\,lb \quad M = 0$


V (lb)
V = 200 - 50x
M (lb ft)
M = 200x - 25x^2



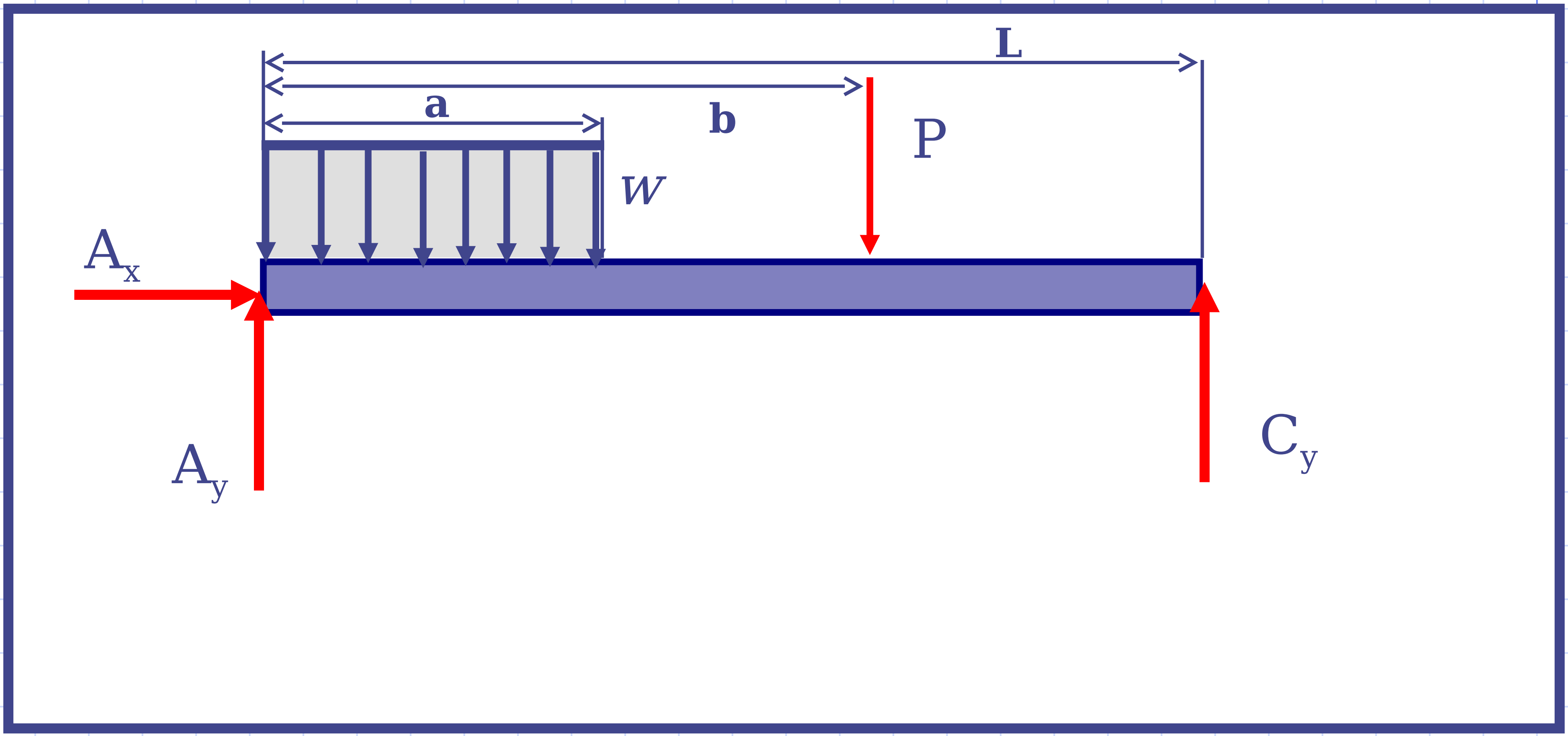
$w = 100$ lb/ft

$L = 15$ ft

$a = 5$ ft

$b = 10$ ft

$P = 1000$ lb



L
a
b
P
w
A_x
A_y
C_y

$$\sum F_x = 0$$

$$A_x = 0$$

$$\sum F_y = 0$$

$$A_y + C_y - 100(5) - 1000 = 0$$

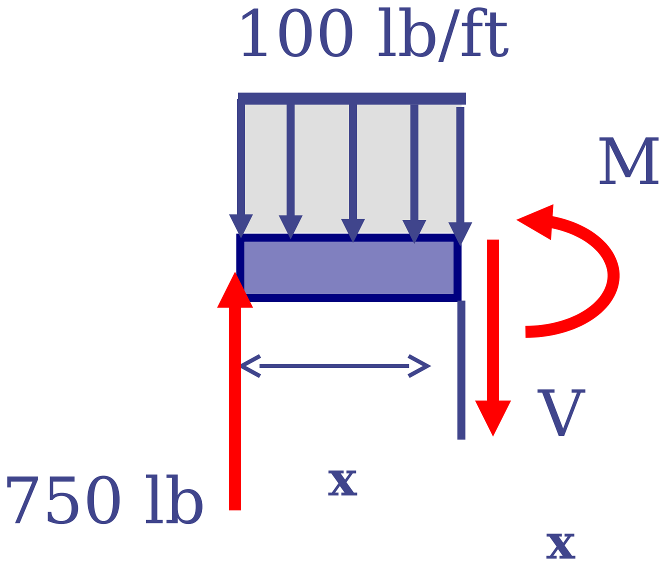
$$A_y + C_y = 1500$$

$$\sum M_A = 0$$

$$-100(5)(2.5) - 1000(10) + C_y(15) = 0$$

$$A_y = 750 \text{ lb}$$

$$C_y = 750 \text{ lb}$$



$$0 \le x \le 5$$

$$\sum F_y = 0$$

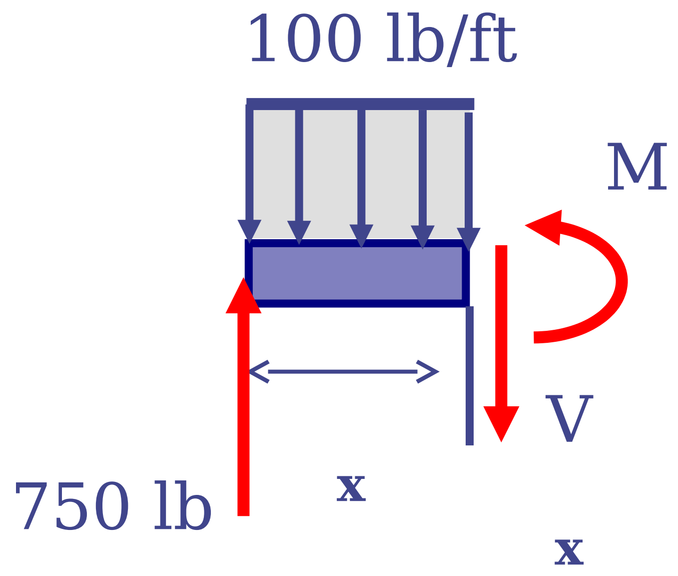
$$-V + 750\ \text{lb} - 100x = 0$$

$$V = (750 - 100x)\ \text{lb}$$

$$\sum M = 0$$

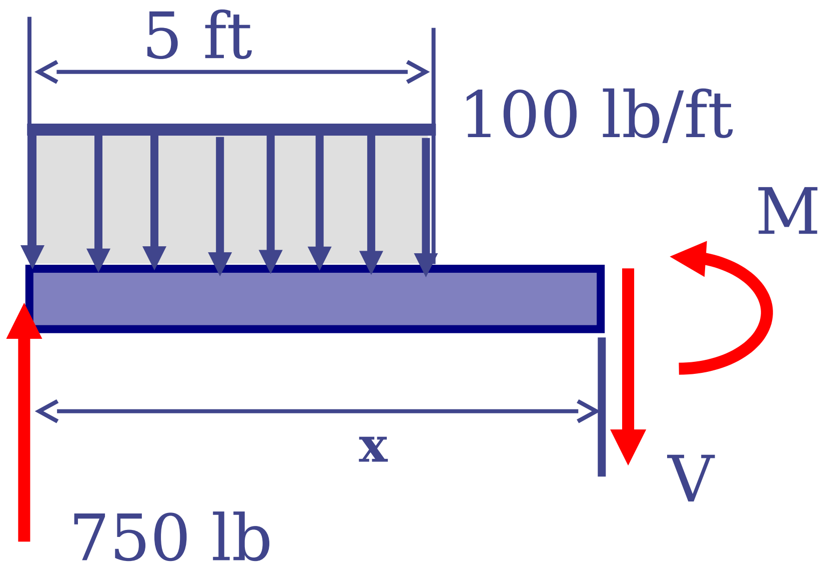
$$M - 750x + 100(x)\left(\frac{x}{2}\right) = 0$$

$$M = \left(750x - 50x^2\right)\ \text{lb}\cdot\text{ft}$$



$$at\, x = 0, \quad V = 750\, lb \qquad M = 0$$

$$at\, x = 5, \quad V = 250\, lb \qquad M = 2500\, lb \cdot ft$$



$$5 \le x < 10$$

$$\sum F_y = 0$$

$$-V + 750\ \text{lb} - 100(5) = 0$$

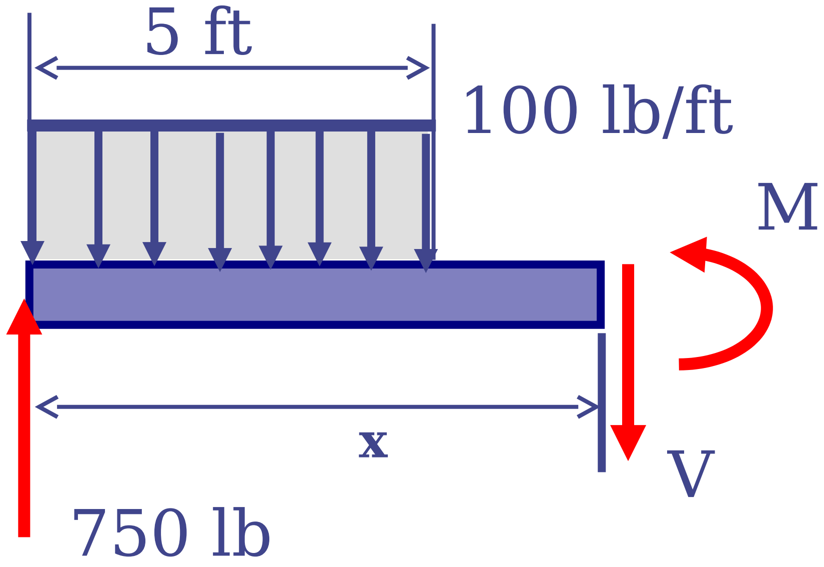
$$V = 250\ \text{lb}$$

$$\sum M = 0$$

$$M - 750x + 100(5)(x - 2.5) = 0$$

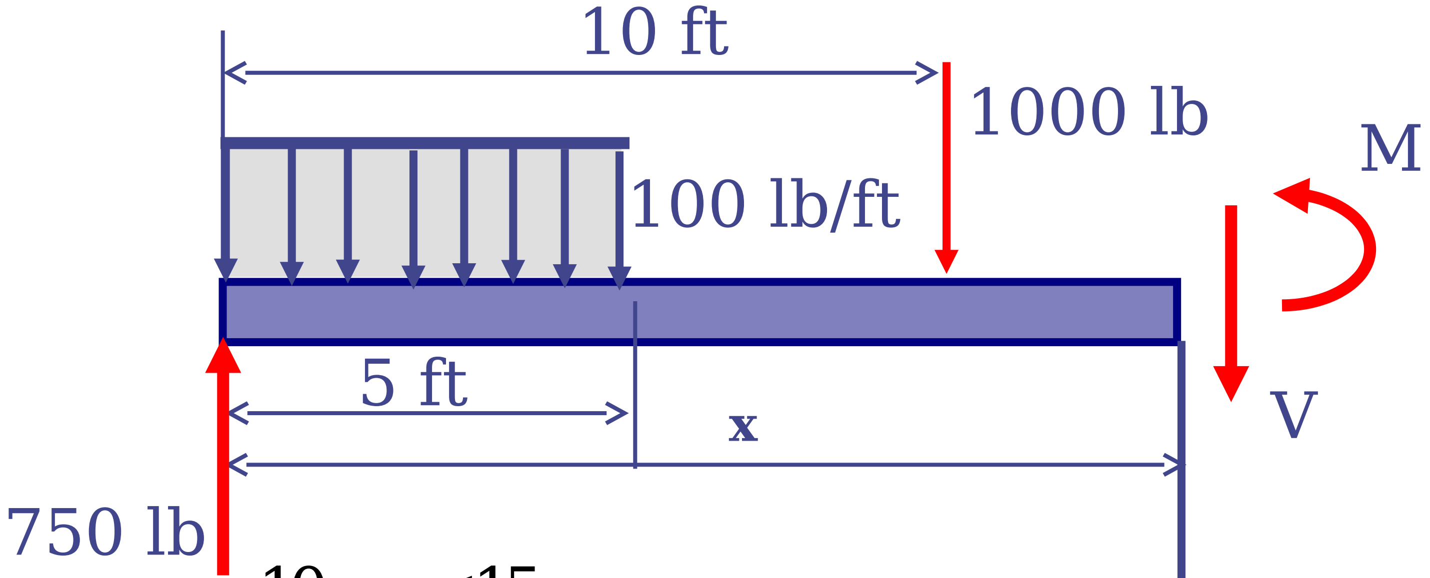
$$M = (750x - 500x + 1250)\ \text{lb}\cdot\text{ft}$$

$$M = (250x + 1250)\ \text{lb}\cdot\text{ft}$$



$$at\,x = 5, \quad V = 250\ lb \qquad M = 2500\ lb \cdot ft$$

$$at\,x = 10, \quad V = 250\ lb \qquad M = 3750\ lb \cdot ft$$



$$10 < x \leq 15$$

$$\sum F_y = 0$$

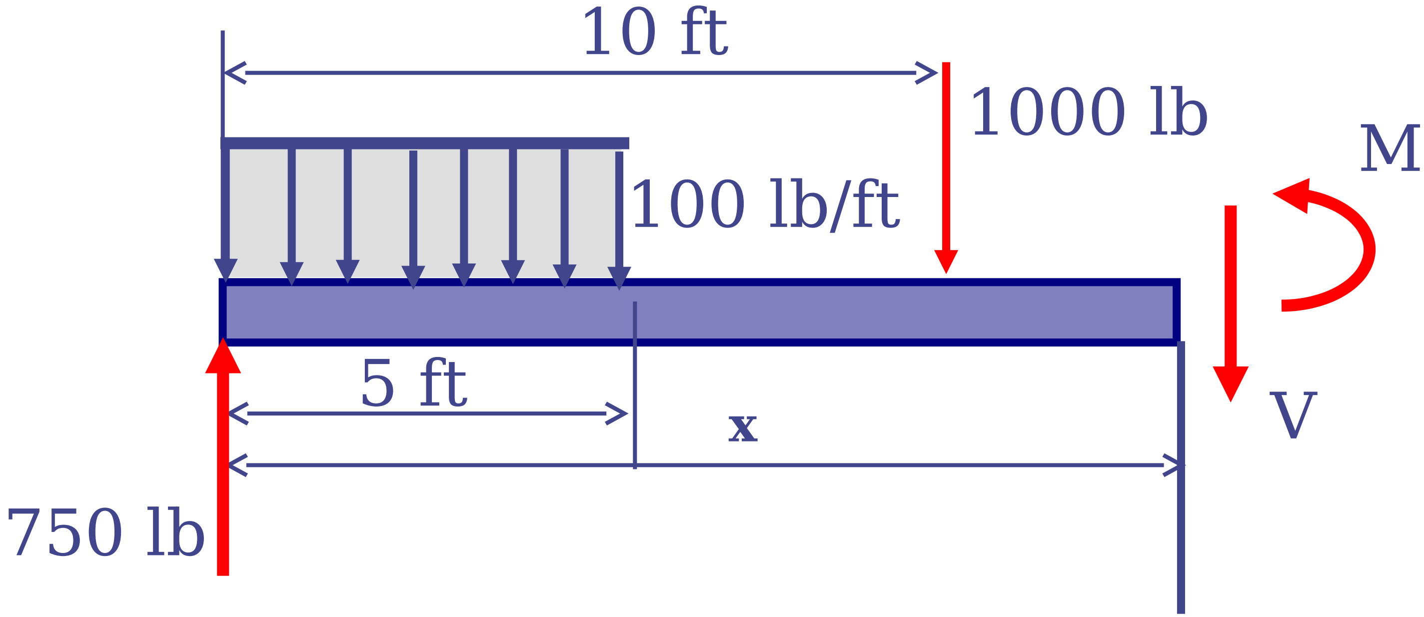
$$-V + 750\ \text{lb} - 100(5) - 1000 = 0$$

$$V = -750\ \text{lb}$$

$$\sum M = 0$$

$$M - 750x + 100(5)(x - 2.5) + 1000\,(x - 10) = 0$$

$$M = (-750x + 11250)\ \text{lb} \cdot \text{ft}$$



$$at\ x = 10^{+},\quad V = -750\ lb \quad M = 3750\ lb\cdot ft$$

$$at\ x = 15,\quad V = -750\ lb \quad M = 0\ lb\cdot ft$$

$$0 \le x \le 5$$

$$V = 750 \text{ lb} - 100x$$

$$M = 750x - 50x^2 \text{ lb}\cdot\text{ft}$$

$$5 \le x < 10$$

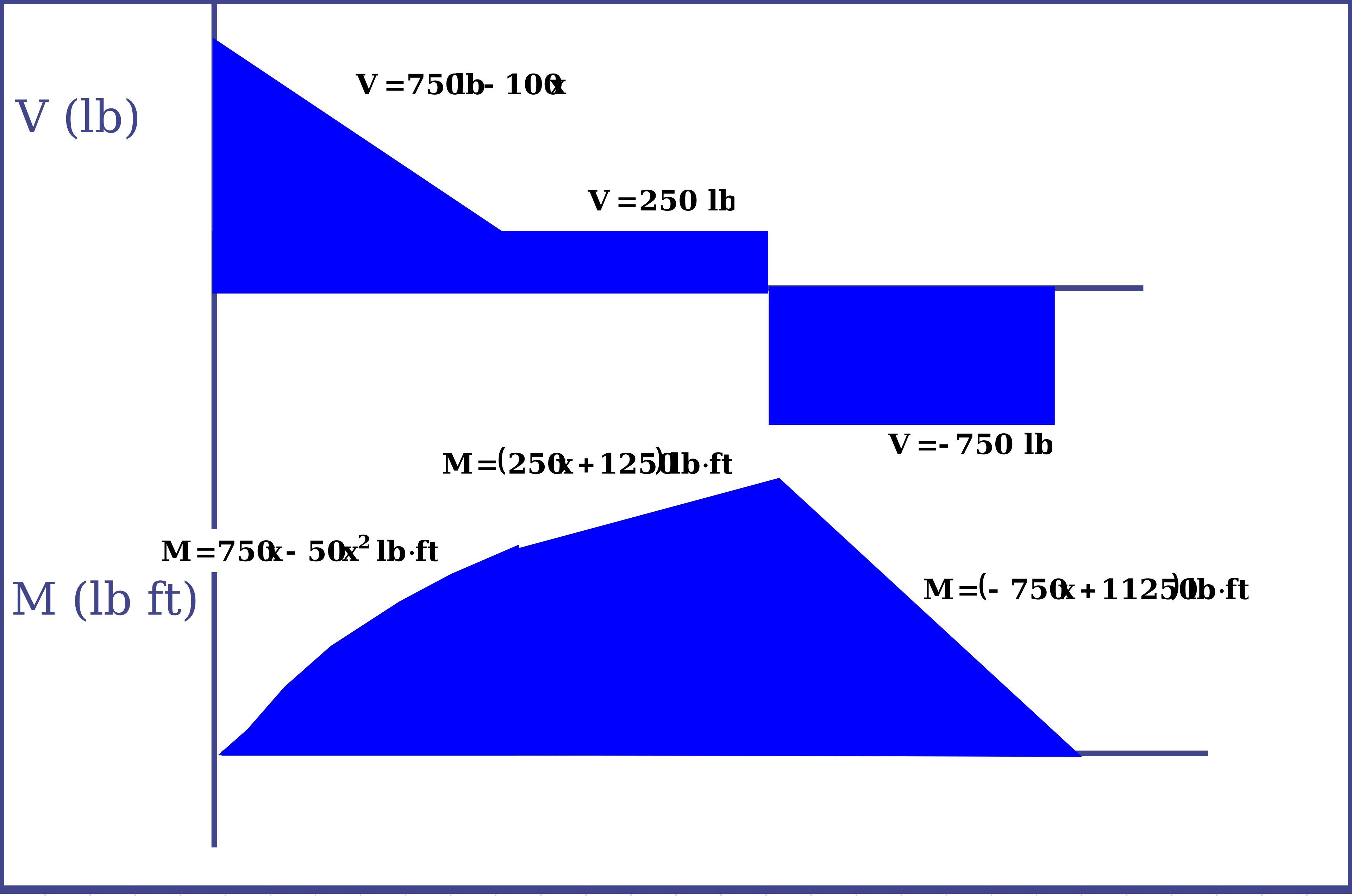
$$V = 250 \text{ lb}$$

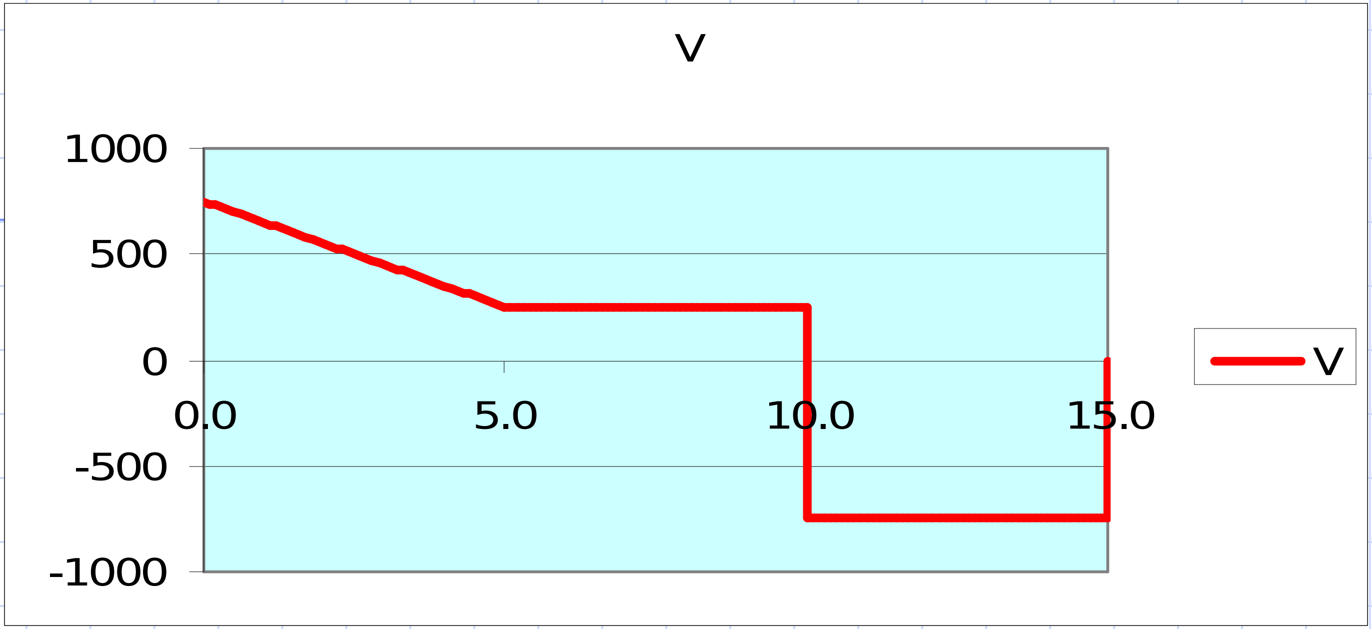
$$M = (250x + 1250) \text{ lb}\cdot\text{ft}$$

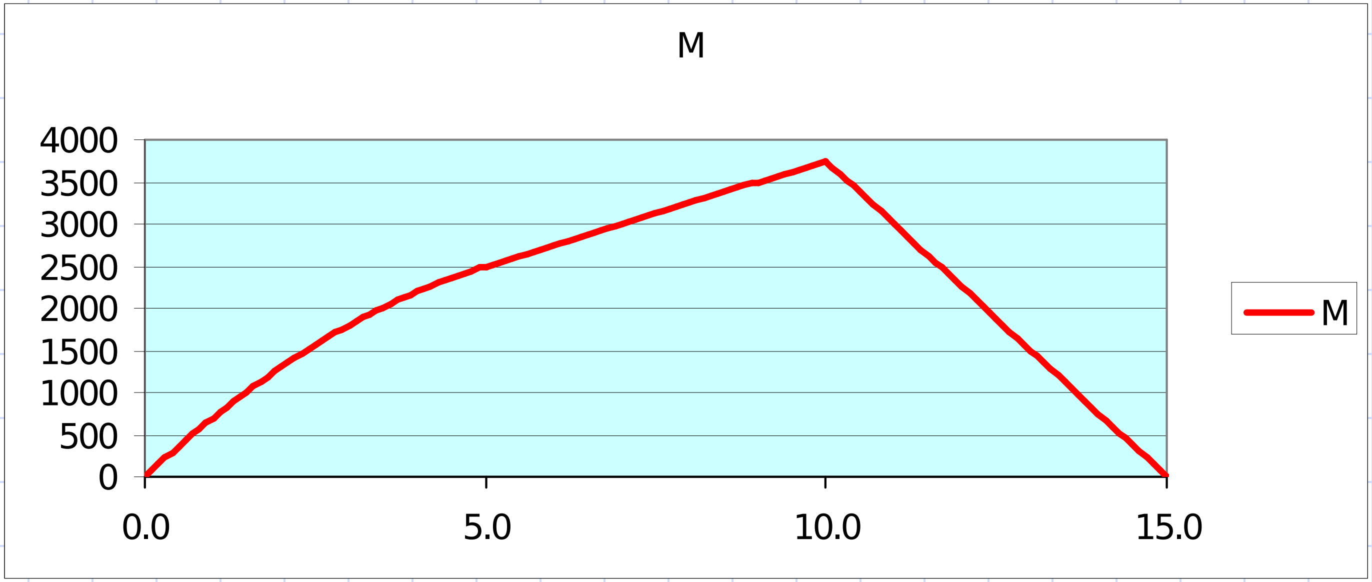
$$10 < x \le 15$$

$$V = -750 \text{ lb}$$

$$M = (-750x + 11250) \text{ lb}\cdot\text{ft}$$


V (lb)
V = 750 lb - 100 x
V = 250 lb
V = - 750 lb
M = (250 x + 1250) lb·ft
M = 750 x - 50 x^2 lb·ft
M (lb ft)
M = (- 750 x + 11250) lb·ft


V
1000
500
0
-500
-1000
0.0
5.0
10.0
15.0
V


M
4000
3500
3000
2500
2000
1500
1000
500
0
0.0
5.0
10.0
15.0
M

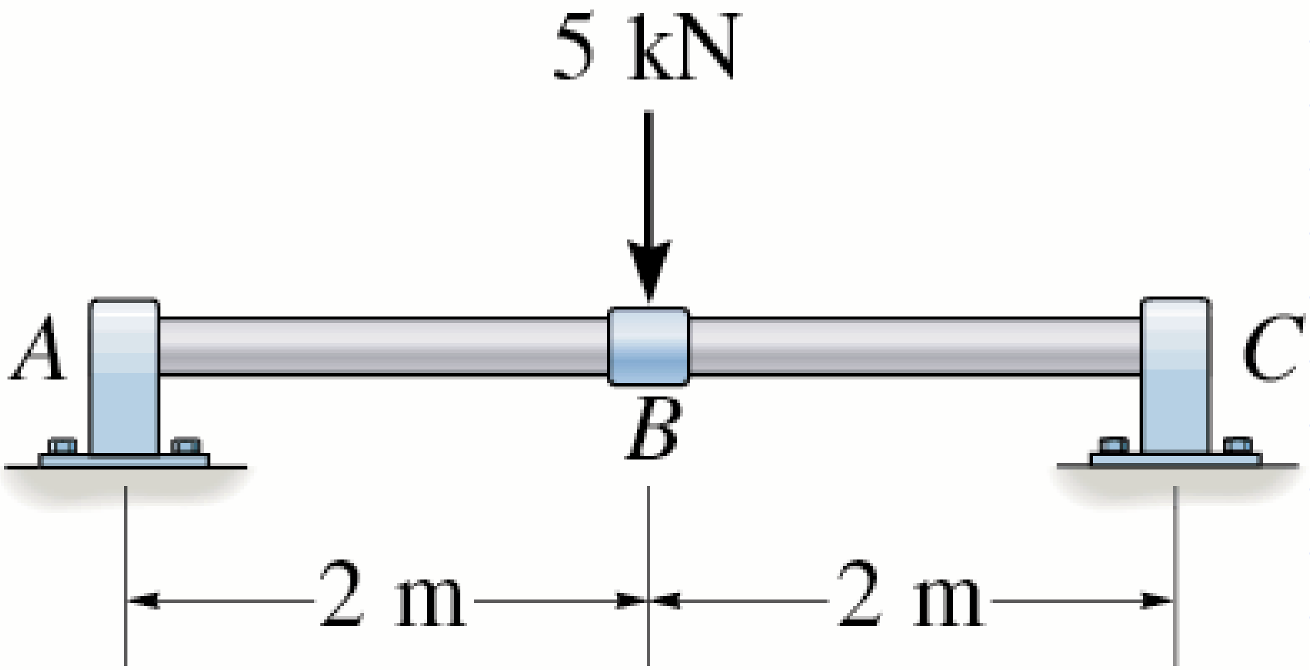


Figure 07.12(a)

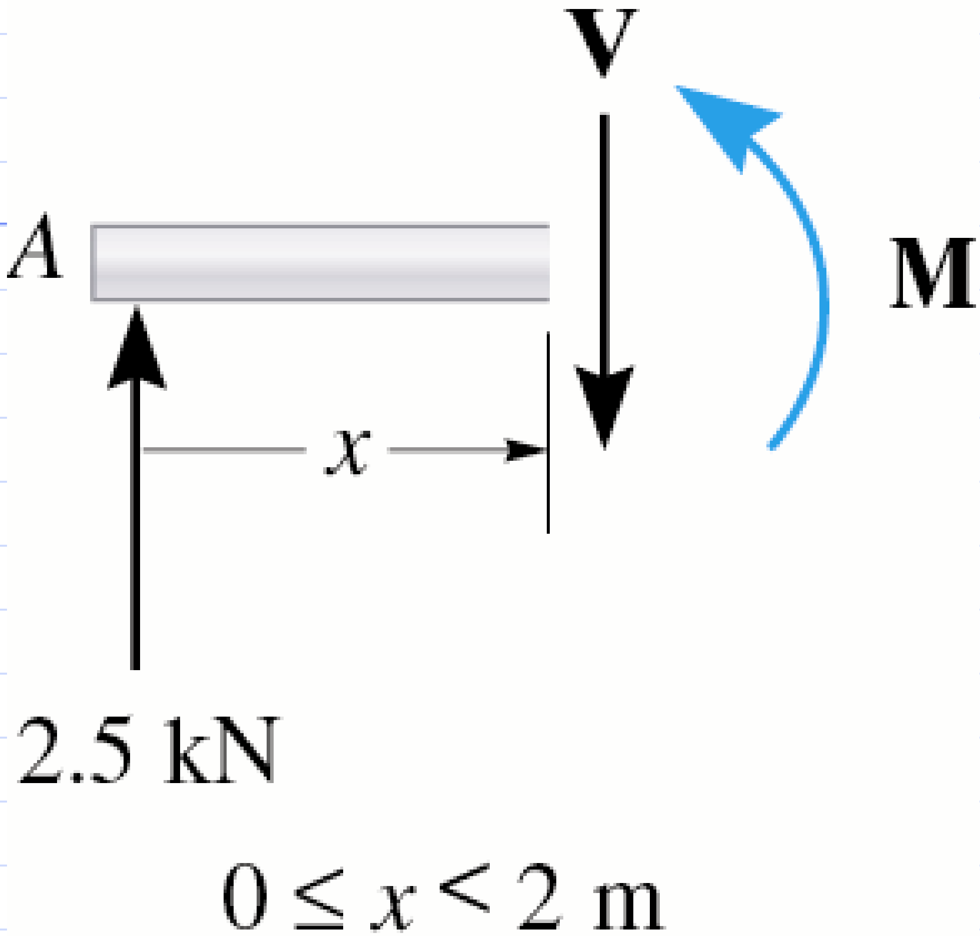
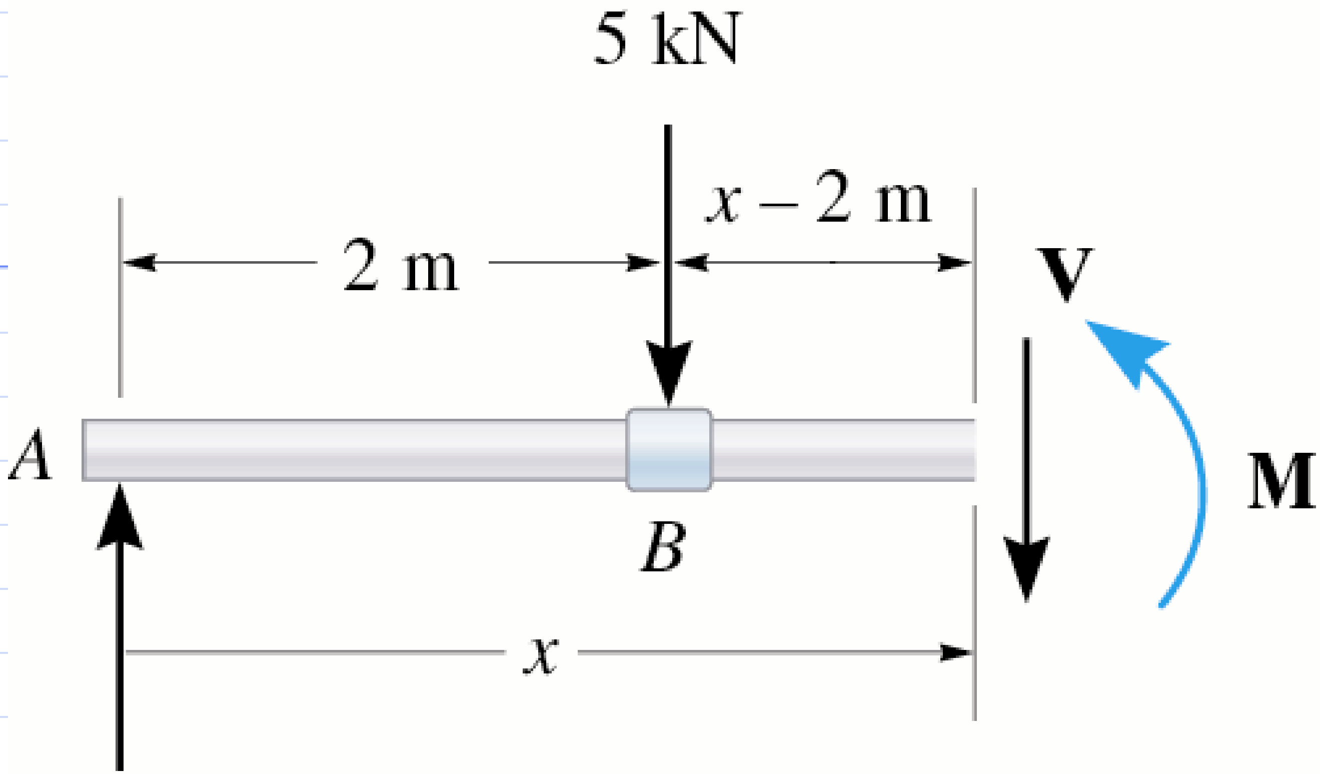
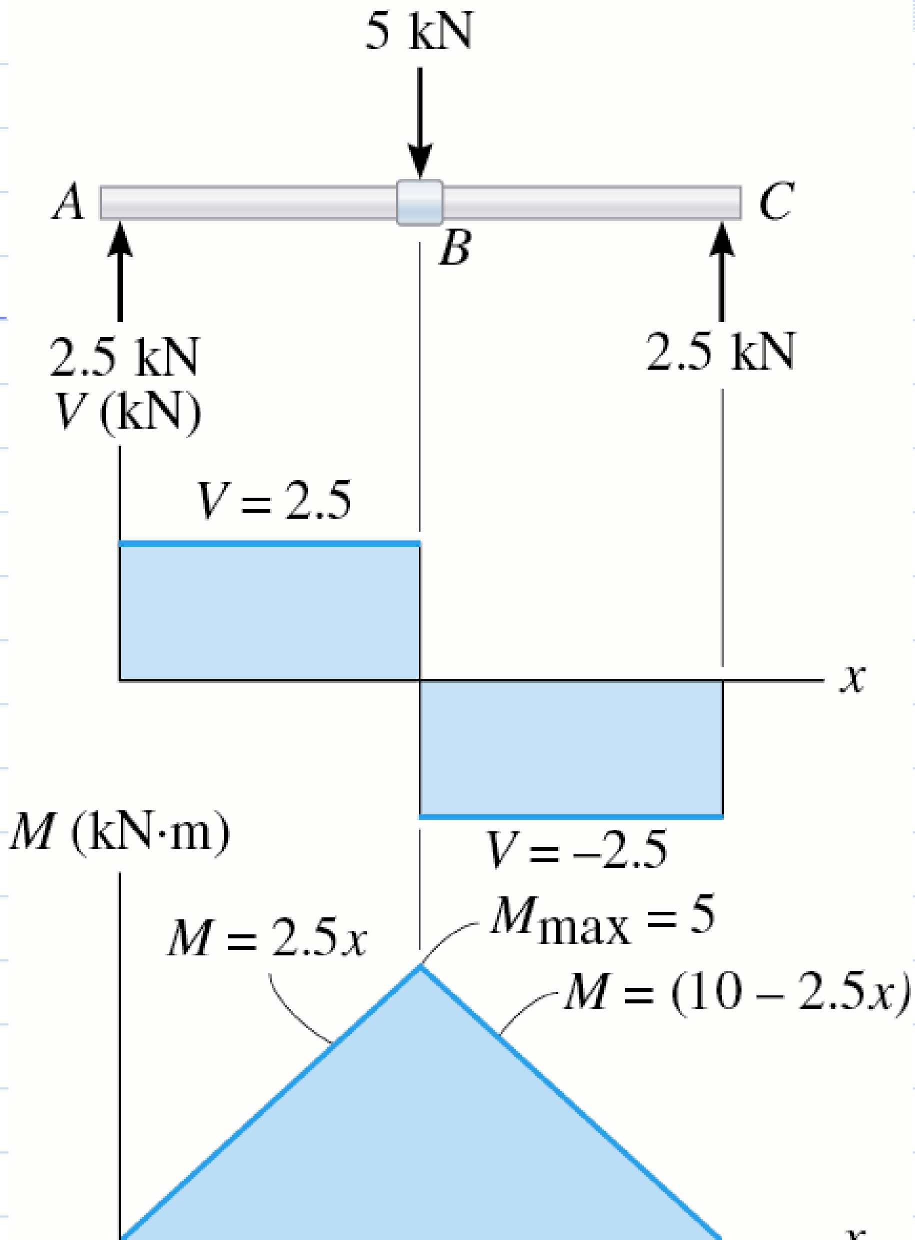


Figure 07.12(b)



$2\text{ m} < x \leq 4\text{ m}$

Figure 07.12(c)


5 kN
A
B
C
2.5 kN
2.5 kN
V (kN)
V = 2.5
x
V = –2.5
M (kN·m)
M = 2.5x
Mmax = 5
M = (10 – 2.5x)
x

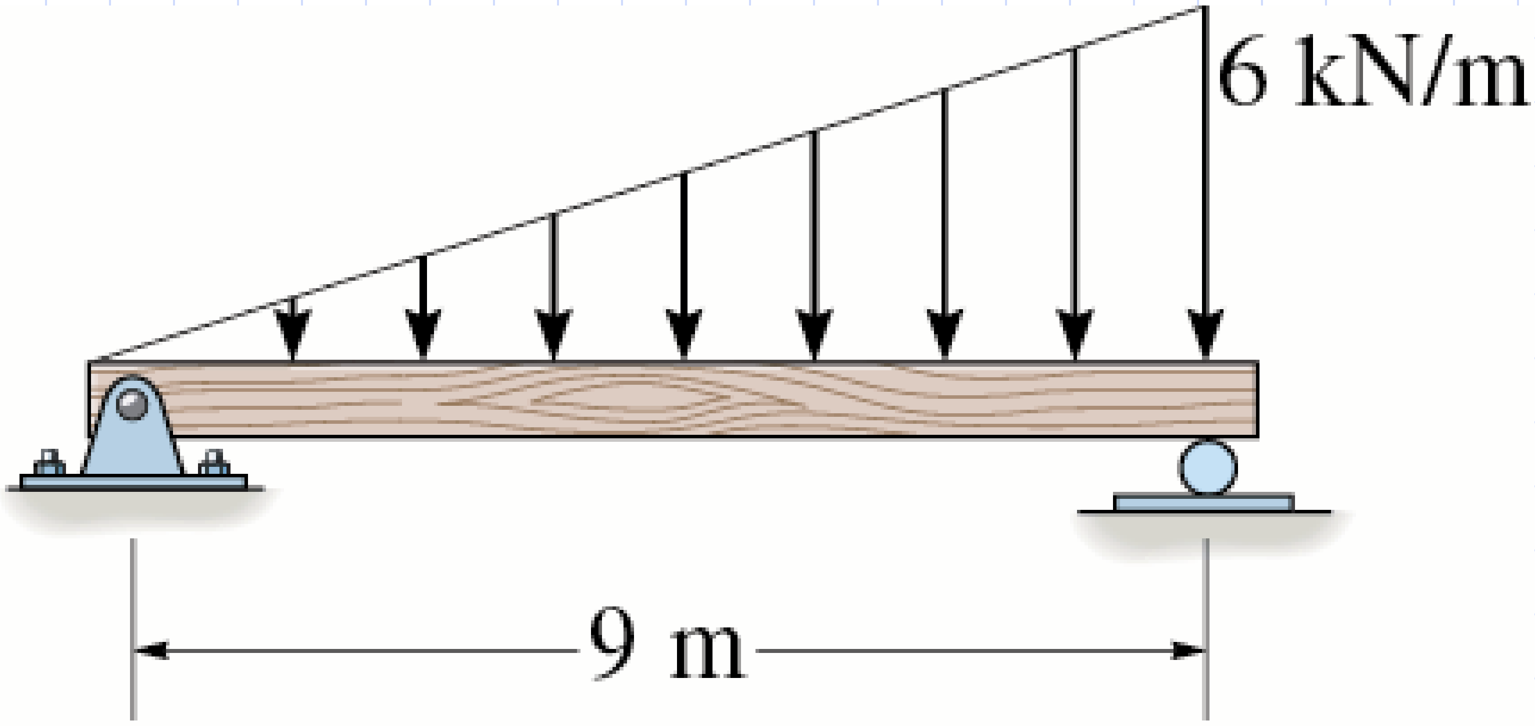


Figure 07.13(a)

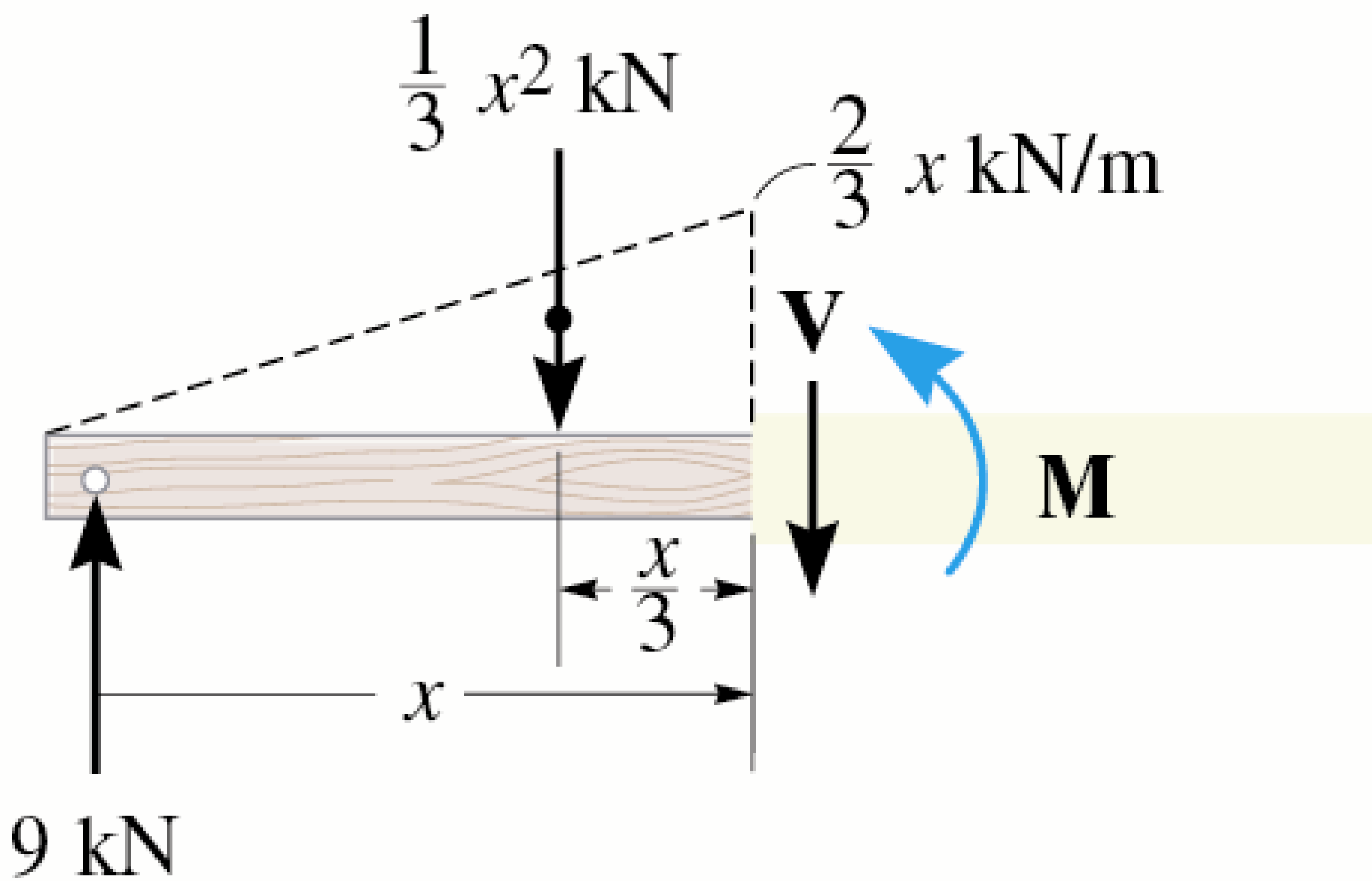
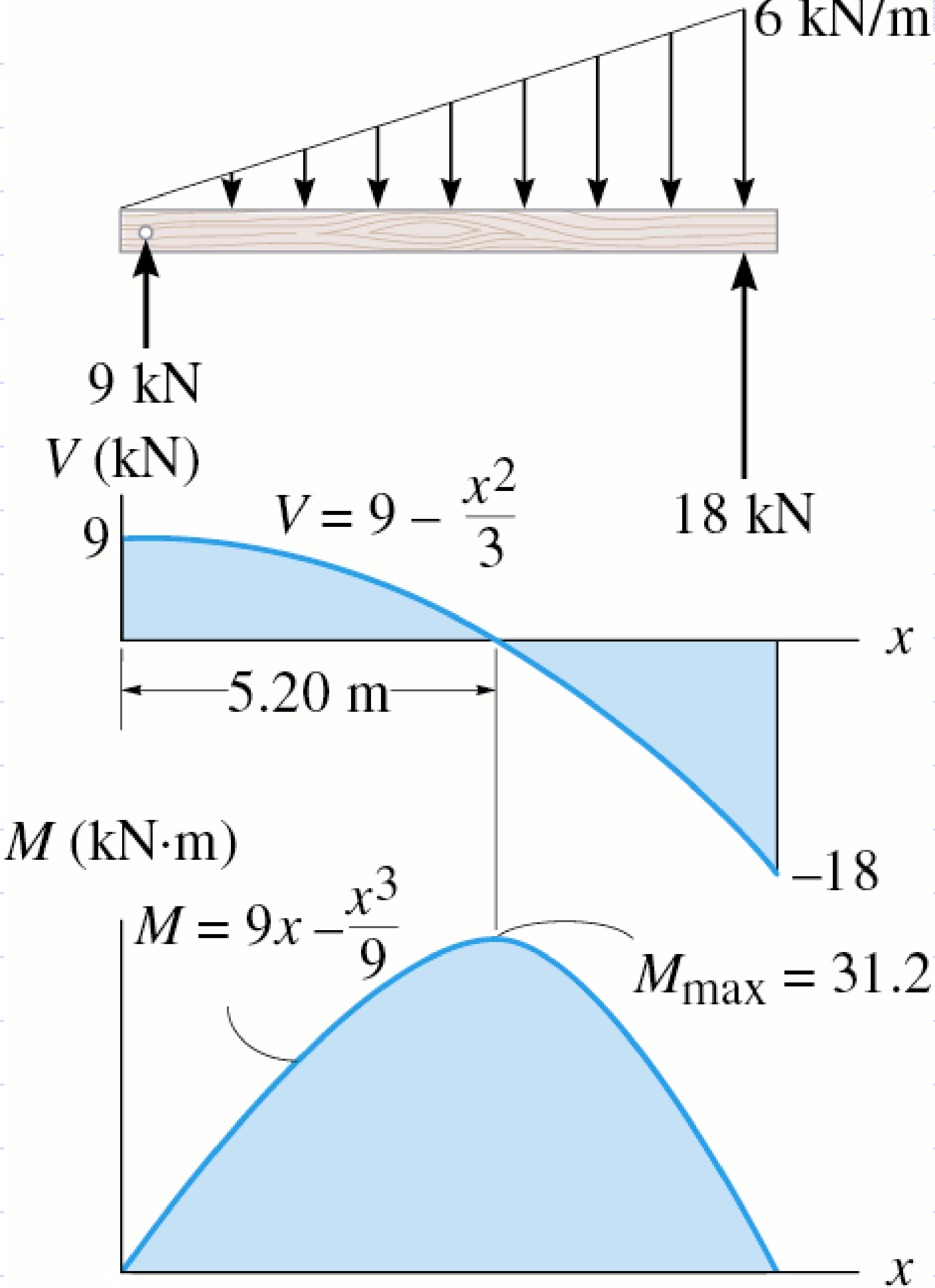
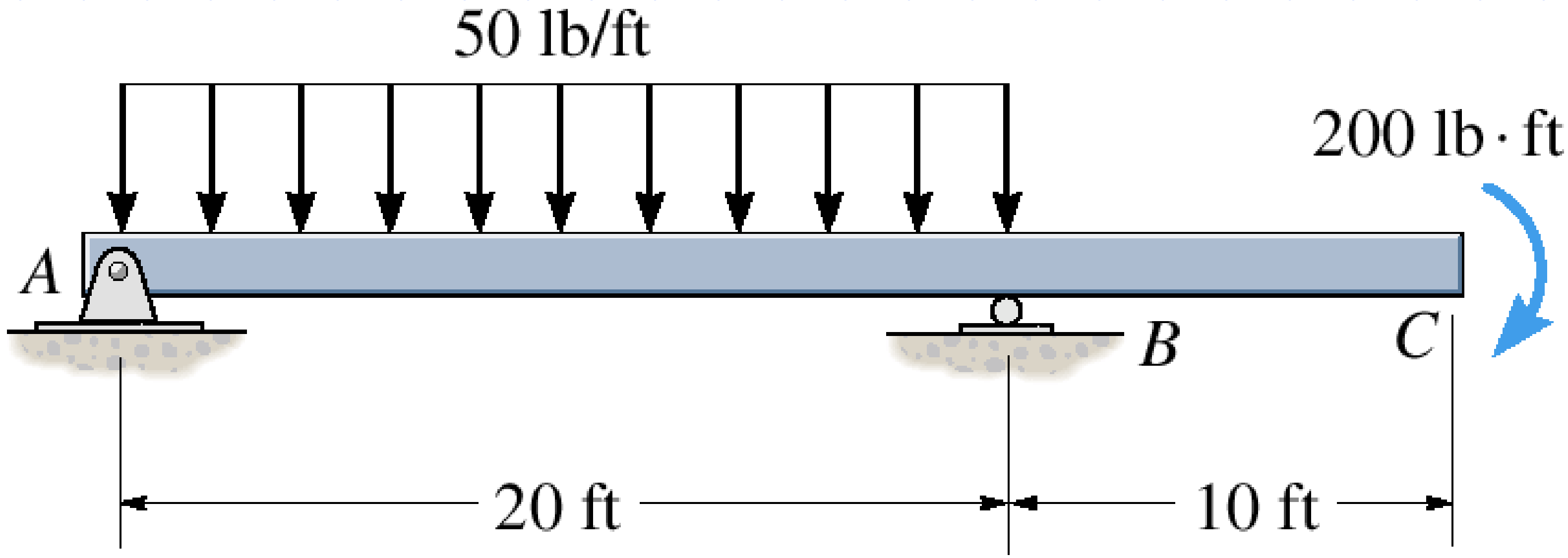


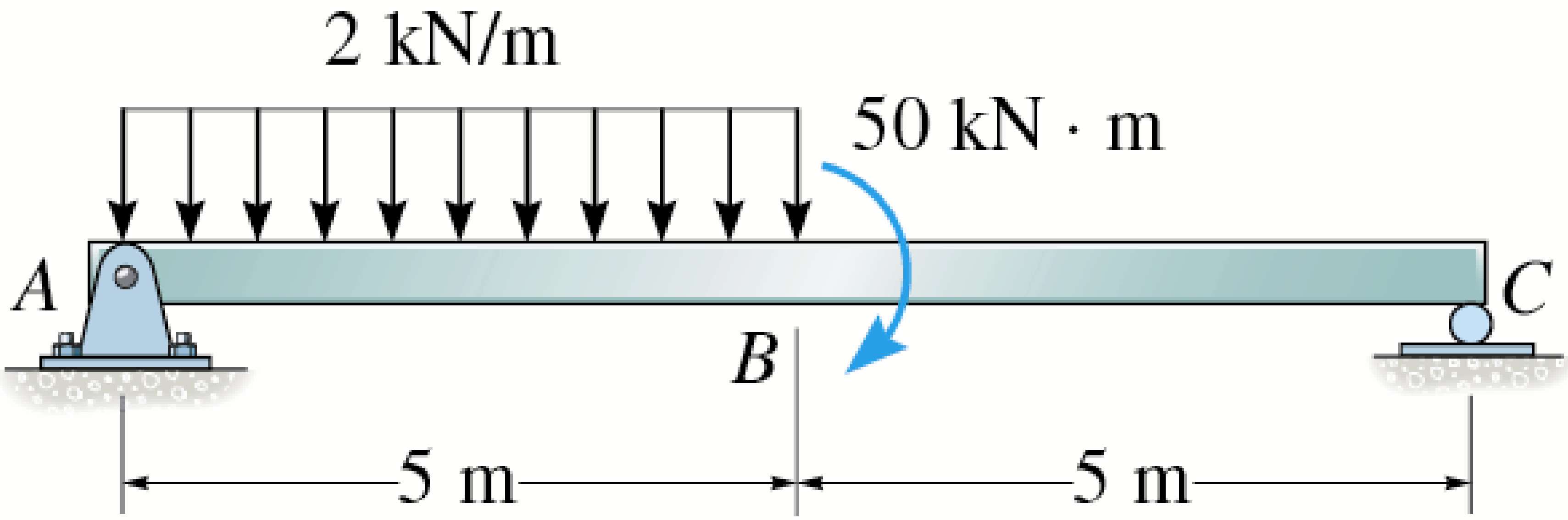
Figure 07.13(b)



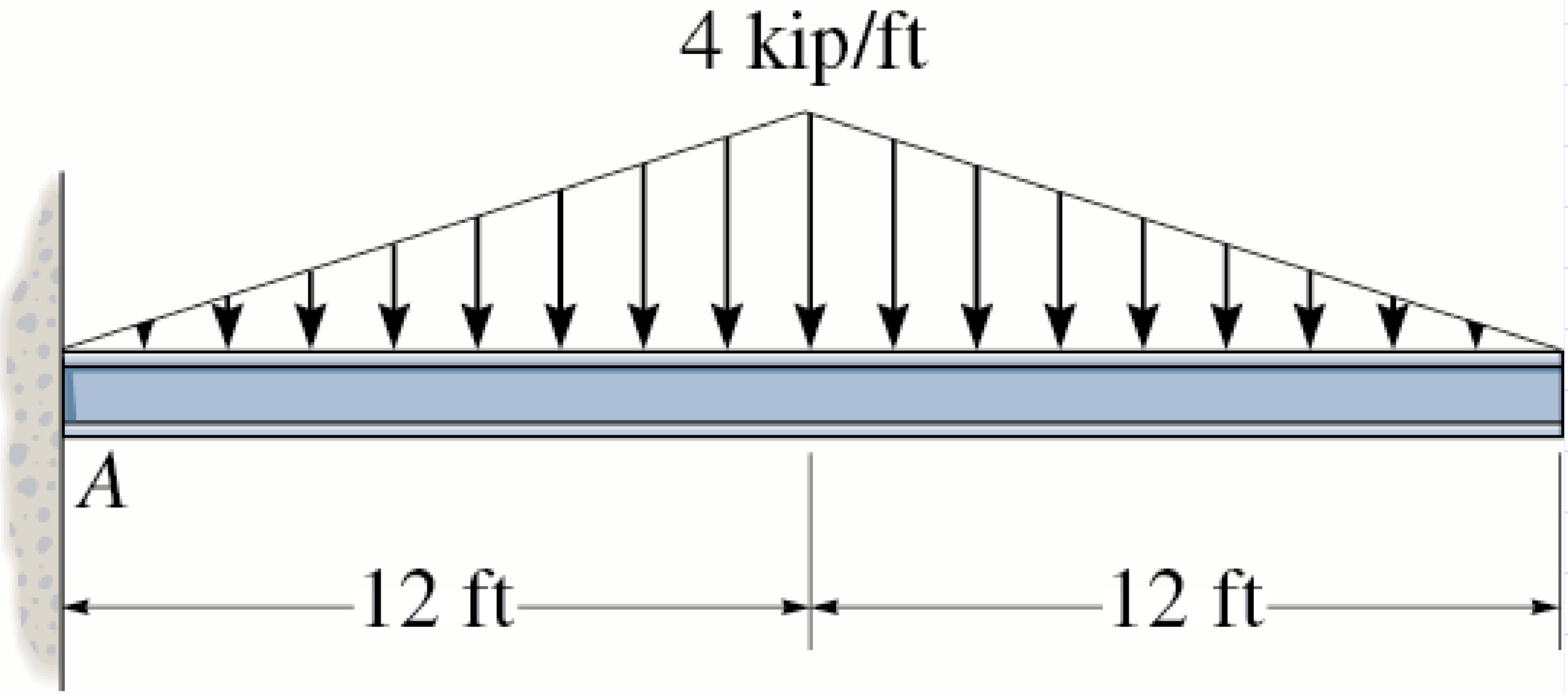
6 kN/m
9 kN
18 kN
V (kN)
9
$V = 9 - \frac{x^2}{3}$
x
5.20 m
–18
M (kN·m)
$M = 9x - \frac{x^3}{9}$
$M_{max} = 31.2$
x



Prob 07.49



Prob 07.51



Prob 07.59